Rethinking Distribution: Introducing Market Segmentation as a Policy Instrument*

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Abstract

Inequality and the skewed distribution of 'essential' goods remain pertinent problems today. Here we consider a general equilibrium framework with essential and non-essentials, where only essential goods are deemed relevant to distributional concerns. We then compare the efficacy of four policies for a utilitarian planner, namely taxation with direct transfers, taxation with subsidies, quantity rationing, and a fourth policy which we introduce and term *market segmentation (MS)*. Under MS, the market for essentials is segmented from non-essentials *(i.e., they cannot be freely traded for each other)*. We find that if the relative number of low-income individuals in the economy is large and "essentials" are consumed inelastically, then MS outperforms taxation with transfers and taxation with subsidies. MS also weakly dominates quantity rationing. We discuss how market segmentation can help policymakers deal with issues such as automation and the superstar phenomenon (Scheuer and Werning, 2017).

Keywords: public economics, public finance, revenue, subsidies, taxation, transfers, rationing, distribution and welfare.

JEL classification: D3, D6, H2.

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1 Introduction

The late 20th and early 21st centuries saw a dramatic increase in wealth and income inequality across the western world (Piketty, 2014). Strikingly, many can no longer consume what most would agree are "essential" goods. An example of this phenomenon is an increase in the number of Americans who are unable to access adequate healthcare.[1] An increase in the prices of essentials, such as housing and healthcare, which crowds out the poor, may well be exacerbating the issue.[2] In this paper, we argue that policymakers must confront this reality by expanding the set of redistributive policy instruments traditionally available to the state.

We consider a stylized general equilibrium exchange economy in which some goods are considered essential and others not. We assume that only essential goods are relevant to distributional concerns. We find that, under certain conditions, conventional redistributive instruments cease to achieve their social welfare goals effectively. We then introduce a new policy instrument called market segmentation (MS hereafter) and demonstrate its superior performance to conventional instruments under these conditions.

Under MS, not every good *type* is freely tradable for others. As we are concerned with distributional issues, we segment the market for essentials from non-essentials, or luxuries *(i.e., they cannot be freely traded for one another)*. We formalize this by not allowing agents to spend income generated by selling non-essential goods on essential goods above a certain threshold level.[3] Thus, we have one budget constraint for essential goods and one for luxuries. In such a setup, we compare the welfare effects of MS with three other policies, namely: 1) taxation with subsidies, 2) taxation with direct transfers, and 3) quantity rationing.

We find that, if the relative number of low-income individuals in the economy is high and "essentials" are consumed in-elastically, then MS outperforms taxation with transfers and taxation with subsidies.[4] The main intuition for this result is that, by disallowing trade between essential and non-essential goods, MS effectively lowers the price of essential goods for the poor. Importantly, it does so without creating inefficiency or dead-weight loss in the trade of non-essential goods *(as there is no tax on trade between non-essential goods)*. In juxtaposition, taxation is inefficient when the economy has a relatively large number of economically disadvantaged because the tax revenues generated are low and raising taxes creates dead-weight losses in an economy. Moreover, when the

[1] A 2019 poll by Gallup found that the number of people putting off healthcare due to affordability has increased from 12% to 25% (Saad, 2019).

[2] For instance, see the trend of house prices in New York City at https://furmancenter.org/files/Trends_in_NYC_Housing_Price_Appreciation.pdf. For healthcare see Nunn et al. (2020).

[3] In our model, there is one non-essential good which is equally endowed to all agents in the economy (referred to as universal goods). These goods are segmented within the essential goods, thereby preventing their free trade with all other luxuries.

[4] Inelasticity means that the price elasticity of demand for essential goods is always less than 1.

demand for essentials is inelastic, taxation fails to significantly decrease the demand from the rich and instead leads to an increase in the equilibrium price.[5]

We then demonstrate that MS outperforms quantity rationing when agents' preferences are heterogeneous. MS makes the agents consume the essentials they prefer. On the other hand, the rich tend to consume the maximum of all essential goods under quantity rationing, thus hampering equal distribution. MS thus exploits the Walrasian equilibrium's "self-selection" properties when individuals have heterogeneous preferences over essential goods to increase welfare for the poor. We defer to Weitzman (1977): "Other things being equal, the price system has greater comparative effectiveness in sorting out the deficit commodity and in getting it to those who need it most when wants are more widely dispersed."

We also show that MS and direct transfers complement each other and that MS and direct transfers should (almost always) be used together. This expands the set of policy instruments available to the state. Fixing the segmented categories, we characterize the optimal segmentary tax in our framework.[6] Further, we consider various applications of segmentary taxes and how they can help in dealing with emergent issues arising due to rapidly changing technological and labor market conditions, such as automation (Costinot and Werning, 2020) and the superstar phenomenon (Scheuer and Werning, 2017).

Importantly, to implement MS, the planner need not know individual endowments or preferences, only their distributions across society. However, to enforce segmentation, the planner must stop the rich from providing side payments to owners of essential goods. Thus, the additional information that the planner would need to implement MS is to be able to link and monitor an individual's expenditure on essential goods. This can be done by linking social security, or unique identification numbers, to expenditure on essentials.[7] We discuss further implementation issues in the main section of the paper.

We introduce segmentary taxes as a policy instrument and provide sufficient conditions under which MS dominates transfers and subsidies. We concede that this is just a small step in finding the general solution to the problem. However, we hope that the reader understands the following salient conclusions, namely that MS does better than transfers and subsidies when; i) essential goods are inelastic with respect to price; ii) there are a relatively large number of poor in the population; and iii) taxes effectively decrease trade within non-essential goods. Further, if needs are heterogeneous, then MS

[5]Even if tax revenue is used to subsidise essential goods, price inelasticity ensures that subsidies don't increase the supply at equilibrium, only prices.

[6]The kind of taxes which we call segmentary do exist in real-world settings, though they have not yet been theoretically explored as a policy instrument. One example of such a tax would be the capital gains tax in India. See (https://cleartax.in/s/capital-gains-income) and many other countries, including the US; (See, Matthew Frankel (2017-12-22)). "Your Guide to Capital Gains Taxes in 2018"

[7]Such linkage of the individual to consumption is already done as part of many social security schemes throughout the world such as the public distribution system in India (see https://nfsa.gov.in/portal/PDS_page).

dominates quantity rationing. We then demonstrate how the optimal utilitarian policy may depend on the nature of change within labor markets.

Our results depend on the price inelasticity of essential goods in terms of both demand and supply. When it comes to demand, we think of this as a natural condition.[8] On the supply side, this may represent institutional constraints or fixed factors for producing essential goods. For example, many studies point out that it is difficult to train more doctors because of technological constraints.[9] Similarly, shortages in land, in the case of urban housing, make supply constraints real even in the long run.[10]. Hence, we think that market segmentation can be a very useful tool in the repertoire of a policy maker in many instances.

We conclude this section with a discussion of the related literature and then, in Section 2, we present a simple example that brings forth the main argument of this paper. Section 3 presents the main model and results. Section 4 discusses applications of the taxes, including their implementation, in greater detail. Finally, Section 5 concludes.

1.1 Related Literature

Our paper contributes to the already extensive literature on redistributive approaches in public finance. Traditionally, economics has dealt with the problems of redistribution via two broad approaches. The first is taxation, pioneered by Ramsey (1927), Diamond and Mirlees (1971), whereby the planner collects tax revenue to facilitate direct transfers or subsidise goods. The second approach is the rationing of essential goods through direct quantity controls (Tobin, 1970), price controls (Weitzman, 1977), or non-linear taxes (Gadenne, 2020). Segmentary taxes combine the above two approaches by employing a tax regime which differentiates between different types of goods (in particular, essential and non-essential goods) so as to ration essential goods by lowering their prices. Thus, it is essentially a 'tax instrument' of price control, one that can avoid dead-weight loss and achieve rationing, after Weitzman (1977), Bulow and Klemperer (2012), Condorelli (2013), and Dworczak et al. (2020).

[8]For our results to hold, the price inelasticity of demand and supply of "essentials" is an important condition. Research estimating the price elasticity of demand on goods such as housing and healthcare finds these to be inelastic. Hanushek and Quigley (1980) estimate price inelasticity of demand housing to be -0.64 in Pittsburgh and -0.45 in Phoenix. Recent papers further substantiate their claims. Albouy, Ehrlich, and Liu (2016) state, *"Since 1970, housing's relative price, share of expenditure, and unaffordability have all grown"*. For healthcare, Ellis, Martins, and Zhu (2017) find elasticity estimates to be between -0.17 and -0.35, even lower than those for housing demand. Also, the elasticity of supply of these goods is low in the short run and is estimated to even be falling in the long run (See: Aastveit, Albuquerque, and Anundsen (2016)).

[9]https://www.washingtonian.com/2020/04/13/were-short-on-healthcare-workers-why-doesnt-the-u-s-just-make-more-doctors describes the legislative problems in expanding doctor supply.
Levit et al. (2009) summarizes the institutional constraints to expanding physician supply

[10]For example, consider the problems discussed by Indian policymakers (see https://niti.gov.in/indias-housing-conundrum

Our main result also holds true in some particular non-linear labor taxation setups.[11] In Appendix C, we build on the framework of Saez (2002) and combine it with some features of Saez (2004).[12] We demonstrate that MS dominates non-linear labor taxation in such a setup when the demand and supply of essential goods are price inelastic. For ease of exposition, we keep the main body of our paper in the framework of Diamond and Mirlees (1971). More generally, there is a vast literature on how non-linear taxes on labor can affect distribution (see Atkinson and Stiglitz (1976); Stantcheva (2014); Piketty et al. (2014); Saez and Stantcheva (2016); Stantcheva (2020)). We consider combining non-linear taxes with segmentation a promising direction for future work. However, in this paper, we constrain ourselves to discussing linear commodity taxation, particularly because nonlinear taxation can usually be replicated by linear taxation (Scheuer and Werning, 2016).

Our paper also relates to that literature which considers how to design markets for specific commodities (for a detailed review of the literature, see Roth (2015); Vulkan et al. (2013)). Many of these 'commodities', such as organs or places in educational institutions, are effectually segmented from the rest of the economy for either moral or egalitarian reasons.[13] In this work, we venture an economic rationale for this segmentation. However, if the planner is utilitarian, we further show that these markets may benefit from limited interactions with others. Moreover, much of this literature treats supply as exogenous and only considers the allocation problem (Dworczak et al., 2020; Akbarpour et al., 2020, 2021). In our paper, we endogenize supply and show how combining certain types of commodities *(essentials)*, while segmenting them from others *(non-essentials)*, can improve welfare.

When discussing applications, we interface with the work on **taxation and redistribution with automation** previously considered by Guerreiro et al. (2017); Costinot and Werning (2020); and Thummel (2020). In the framework of Guerreiro et al. (2017), technical progress in automation and endogenous skill choice can lead to a massive rise in income inequality. Costinot and Werning (2020) use a sufficient statistic approach to characterize an optimal tax on robots. Thummel (2020) provides the closest parallel to our work because of the discussion of the general equilibrium effects of taxing robots which, in turn, changes relative prices and, thereby, income distribution. Our paper departs from the preceding work in a few crucial aspects. First, we distinguish between essential and inessential goods, highlighting the possibility of segmenting these markets to enhance welfare. Second, we show that different policies are more efficient at increasing welfare under the different effects of automation that we consider.

[11] Non linear labor taxation was pioneered by Mirrlees (1971).

[12] Though Atkinson and Stiglitz (1976) show that, if utility is separable between labor and consumption, only labor taxes are sufficient to improve welfare, although Naito (1999) notes that this result does not hold if relative prices in the economy are subject to change.

[13] See Roth (2018)

Dealing with yet other applications, we find that the precise nature of labor markets that produce either 'superstar effects' (Scheuer and Werning, 2017) or 'winner-take-all' (Rothchild and Scheuer, 2016) scenarios makes MS more relevant. MS allows the planner to change the distribution of essential goods without causing excess distortions in production, which is the major issue in the above-mentioned labor markets.[14]

To the best of our knowledge, our paper is the first to introduce segmentary taxes. Thus, although our results are similar in scope and goal to the aforementioned papers, our approach is entirely novel. Importantly, we expand the set of policy instruments available to the state and demonstrate those conditions under which our proposal performs better than existing instruments. The following section provides a simple example that distils our main result's central intuition.

2 Example

A stylized example is now constructed for a clear exposition of the forces behind the main results of this paper. Consider a pure exchange economy with three goods, specifically one essential- x_1 (medicine), and two non-essential goods: x_2 (time/manual labor - universally endowed) and x_3 (gold - luxury good). Going ahead, we call the universally endowed goods as universal goods. The economy is populated by three types of individuals (A, B, and C). The proportion of each type is given by π_A, π_B and π_C, respectively. We assume that all individuals have identical log-linear preferences represented by $U = klogx_1 + x_2 + x_3$.

The utility function captures the intuition that a complete absence of medicines/health services is hugely detrimental to an individual's welfare. Now, assume the planner is utilitarian and wishes to maximize the sum of individual utilities in this economy. We represent his objective function as:

$$W = \pi_A U_A + \pi_B U_B + \pi_C U_c.$$

For the sake of simplicity, let us assume that $\pi_A = \pi_C = \frac{1}{n+2}$ and $\pi_B = \frac{n}{n+2}$. The initial endowments are captured in the Table below.

Agent Type	x_1	x_2	x_3
A (Pharma)	1	e	0
B (Poor)	0	e	0
C (Rich)	0	e	W

Type A are the 'pharmas', *i.e.* those pharmaceutical companies which are endowed with the medicines (and time, of course); whereas type B are the poor who are endowed

[14]For a review on how changing labor market conditions affect optimal taxation policies, see Scheuer and Slemrod (2019).

with *only* time; and type C are the rich who are endowed with both gold and time. Consider a scenario in which the planner can pick between market segmentation (MS) and *laissez-faire* (LF) policies to maximize the social welfare function. Under MS, an agent can only trade between universal (time), and essential goods (such as medicines and health services) and hence cannot use a luxury good (gold) to buy the essential good. Table 1 shows the different budget constraints of an individual under the previously stated 2 regimes.

Table 1: Different Regimes

Regime	Budget Constraint
MS	$\sum_{1,2} p_i(x_i - e_i) \leq 0$ and $p_3(x_3 - e_3) \leq 0$
LF	$\sum_{1,2,3} p_i(x_i - e_i) \leq 0$

In this setup, if the endowment of the universally endowed good is small and the relative number of the poor in the economy is large, then social welfare under MS is higher than under LF.[15] First, note that, due to the quasi-linearity of the utility function, only the distribution of x_1, i.e., essential good is welfare-relevant. MS works by lowering the equilibrium price of x_1, making its allocation more egalitarian by allowing the poor to consume more and thereby increasing social welfare. Figure 1 below demonstrates that, for a low level of endowment of the universal good, welfare under MS is always greater than welfare under LF.

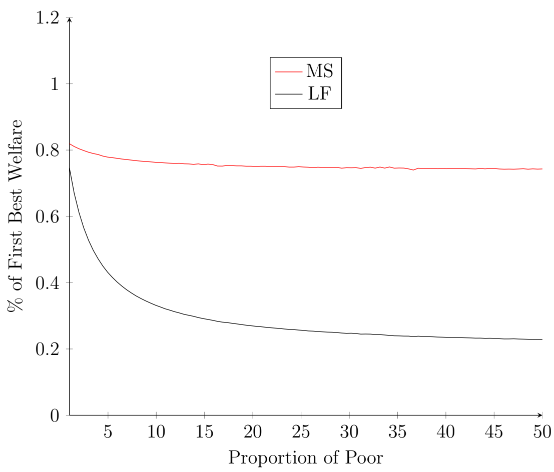


Figure 1: MS vs LF

However, it is worth noting that we can replicate the MS allocation in the above economy by using two pre-existing policy instruments, namely quantity rationing (QR)

[15] To be precise, small endowments of the universal good means that $e << k$, where e is the endowment of the universal good in the economy. For formal setup, see Appendix: Section A.1.

and taxation. First, using QR, which fixes an amount that can be bought for each essential commodity, we could ration the essential good's consumption at the level arising in the MS equilibrium. However, in an economy with more than one essential good and heterogeneous preferences, MS becomes relevant. MS disallows luxury goods to be exchanged for essentials and rations 'jointly', and not good by good, and thereby allows poorer individuals greater freedom in allocating their budgets. At the same time, MS prevents the rich from buying all the available essential goods at the maximum possible levels allowed under quantity rationing. We will examine this in more detail going forward.

Second, within an economy in which there is only one luxury good, taxation weakly dominates both MS and QR. In such a setting, we could simply impose a high tax on the luxury good to reduce its trade with essential goods. At a very high level of tax, no luxury good will be traded with the essential good, thus replicating the MS equilibrium. However, where there is more than one luxury good and heterogeneity in preferences for luxuries, high taxes reduce the trade of these goods, not only for the essential goods, but also for each other. Thus, taxation can potentially be highly distortionary. Here, MS becomes relevant, as it creates an equitable distribution of essential goods while avoiding dead weight losses in the market for luxuries.

Hence, we now move to a setting in which there are two essential goods, two luxury goods, and one universal good. Within this economy we demonstrate that, under certain conditions, MS achieves the objective of the planner more effectively than either QR or taxation.

Five Goods and Preference Heterogeneity

In our 5-good economy, we create an environment in which there exists preference heterogeneity for both essential goods and luxuries. Here, we first show that MS weakly dominates QR when considering the welfare of the poor within the economy. Second, we show that as the number of poor (n) increases, MS eventually dominates taxation with transfers. Therefore, when the number of poor in the economy is large, MS is preferable to both of those policies. We highlight that the levels of taxation required to enforce the allocation of resources which the planner desires are too high and instead create large dead-weight losses. In this scenario, segmentation offers a middle ground.

We have 5 goods in the economy: - 2 essential goods (x_1, x_2), 3 non essentials: x_3 which is universally endowed and 2 luxury goods (x_4, x_5). There are still 3 kinds of individuals:- A (pharmas), B (poor), and C (rich). In C, we have two sub-kinds, i.e., they are endowed with different luxury goods. We assume that the total utility for any agent is additively separable across all goods. The endowments are given in Table 2. Our assumption of heterogeneous preferences makes trade in luxury goods welfare improving.[16]

Table 2: Endowments

Agents	x_1	x_2	x_3	x_4	x_5
A	1	1	e	0	0
B	0	0	e	0	0
C_1	0	0	e	W	0
C_2	0	0	e	0	W

So,

$$U^\omega(X) = \sum_{i=1}^{5} U_i^\omega(x_i)$$

where U_i^ω are given in Table 3. Preferences for essentials are uniform and I.I.D.: $(k_1, k_2)^\omega = \{(0, 2k), (k, k), (2k, 0)\}$ each for a third of the population. We assume that $e < k < \frac{W}{2}$. This implies that the poor only consume essential goods (*i.e.,* are on the boundary), whereas the rich consume both essential goods and luxuries.

Table 3: Utilities

Agents	$U_1(x)$	$U_2(x)$	$U_3(x)$	$U_4(x)$	$U_5(x)$
A	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	$\frac{x}{2}$
B	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	$\frac{x}{2}$
C_1	$k_1^\omega logx$	$k_2^\omega logx$	x	$\frac{x}{2}$	x
C_2	$k_1^\omega logx$	$k_2^\omega logx$	x	x	$\frac{x}{2}$

Table 4 presents budget sets of individuals under different regimes. Note that goods 1 and 2 are the essentials, whereas good 3 is the universal good, and goods 4 and 5 are the luxuries. MS gives the individual two simultaneous budget constraints to satisfy, one for the essential and universal goods corresponding to x_1, x_2, x_3, and the other for luxuries. Under a regime with taxation and direct transfers (DT), individuals receive a lump sum transfer of R and have to pay a tax if they are net sellers of luxuries corresponding to $x_i - e_i < 0$. QR gives individuals one budget constraint but disallows them from buying more than a fixed amount of essentials, as given by limit q_1^*, q_2^*.

When considering equilibrium prices of essential goods, this economy mimics the setup wherein individuals have identical preferences for the two essential goods with coefficient k. This is because of the symmetric nature of the heterogeneity introduced as a uniform I.I.D. distribution. Both essential goods have the same price p.

[16]This is mainly for the case of linearity in non-essential goods. If convex preferences replace linearity, this should not be needed.

Regime	Budget Set
MS	$\sum_{1,2,3} p_i(x_i - e_i) \leq 0$ $\sum_{4,5} p_i(x_i - e_i) \leq 0$
DT	$\sum_{1,2,3} p_i(x_i - e_i) + \sum_{4,5} p_i(1 - \tau \mathbf{I}_{[e_i - x_i]})(x_i - e_i) \leq R$ $\mathbf{I}_{[e_i - x_i]} = 1$ if $e_i - x_i > 0$, $T =$ transfer, and $\tau =$ tax rate.
QR	$\sum_{1}^{5} p_i(x_i - e_i) = 0$ For $i = 2, 3 \quad (x_i - e_1) \leq q_i^*$

Table 4: Budget Sets

Laissez-faire

We solve for the equilibrium allocation without any intervention to serve as a baseline.

Remark. Recall that only the distribution of essential goods is welfare relevant. The price (with respect to the universal good – *i.e.* the *numéraire*) is given by $p^* = \frac{1}{2}ne + 3k$, where n is the relative number of poor in the economy. By construction, $k_1 + k_2 = 2k$. Moreover in a laissez-faire economy, there is no dead-weight loss in the economy. Each essential good (x_1 and x_2) is thus distributed as follows:

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{\frac{1}{2}ne+3k}$	$\frac{k_i^\omega}{\frac{1}{2}ne+3k}$	$\frac{k_i^\omega}{\frac{1}{2}ne+3k}$

Market Segmentation Regime

Under MS, agents can only trade between universal and essential goods and cannot use non-essentials to buy essentials. However, free trade is allowed between non-essential goods. This maximization exercise is similar to the LF case although, since the rich cannot use luxuries to buy health services, their endowments are effectively identical to those of the poor. However, 'pharma' still consumes the essential good at the equilibrium 'satiation level.'

Now again, as before, let $e < k < \frac{W}{2} + 1$. We know, as before, that poor people (B) will demand $x_i^{B,\omega} = \frac{k_i^\omega}{2k}\frac{e}{p_i}$, but now the rich (C) will also demand $x_1^{C,\omega} = \frac{k_i^\omega}{2k}\frac{e}{p_i}$. Moreover, assuming that $m_A = p_1 + 1 > k$, individual A will demand $x_i^{A,\omega} = \frac{k_i^\omega}{p_1}$.

Now each type ω has a different demand. To clear markets, we take expectations over all types $\omega \in \Omega$.

$$\mathbb{E}_{\omega\in\Omega}[x_i^A] + n \times \mathbb{E}_{\omega\in\Omega}[x_i^B] + 2\mathbb{E}_{\omega\in\Omega}[x_i^C] = 1$$

$$\implies p_{MS} = \frac{1}{2}e(n+2) + k = p^* - \overbrace{2(k - \frac{e}{2})}^{\text{Price Effect}}.$$

The price thus goes down when $e << k$. This 'price reduction' increases the welfare of the poor. There is no distortion to the equilibrium allocation of luxuries (*i.e.,* goods 3 and 4 under MS). Each essential good (namely x_1 and x_2) is distributed as follows:

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{p_{MS}}$	$\frac{k_i^\omega}{p_{MS}}$	$\frac{k_i^\omega}{2k}\frac{e}{p_{MS}}$

Quantity Rationing (QR)

The welfare of the poor under MS is at least as favorable as the best possible QR policy. Under QR, the planner sets an upper limit on the quantity of essentials which can be purchased. We find that if this limit is higher than the consumption of the poor under MS, aggregate demand is always greater than the aggregate supply, hence it is not a sustainable equilibrium.

To see this in mathematical terms, suppose the planner sets limit $q^* = max_\omega\{x_{MS}\} = \frac{e}{p_{MS}} =$ and, $p_{QR} = p_{MS}$, trying replicate the MS allocation. Remember that $(k_1, k_2)^\omega = \{(0, 2k), (k, k), (2k, 0)\}$ for each third of the population.[17] Now, the rich with $(k_1, k_2)^\omega = (0, 2k), (2k, 0)$ consume only one good and will be at the rationing limit. However, the rich with $(k_1, k_2)^\omega = (k, k)$ consume $\frac{e}{p_{MS}}$, which is greater than the quantum they consumed under MS - $\frac{1}{2}\frac{e}{p_{MS}}$. This higher demand by a third of the rich leads to excess aggregate demand at this price and, consequently, the market does not clear. Hence, q^* must be reduced and the poor have their welfare diminished.

The best possible quantity rationing equilibrium under this setting has price $p_{QR} = k + \frac{ne}{2} + \frac{4}{3}e > p_{MS}$ and the quantities below. Clearly, the poor are worse off under this policy than MS.

	A	B	C
x_i	$\frac{k_i^\omega}{2k}\frac{e}{p_{QR}}$	$\frac{k_i^\omega}{p_{QR}}$	$\min\{\frac{k_i^\omega}{p_{QR}}, \frac{e}{p_{QR}}\}$

Thus, MS achieves equitable distribution more effectively than QR by addressing the heterogeneity of needs in the economy. With a large number of poor in the economy, this

[17](and for each sub-type, namely the rich, the poor and the pharmas).

translates into an overall increase in social welfare. Section 3 provides a general proof of this phenomenon.

Taxation and Direct Transfers (DT regime)

We now discuss a regime wherein the policymaker can raise revenues by taxing luxuries and providing direct transfers to all individuals in the economy. We show that, when taxes are low, transfers are ineffective because revenues are insufficient. Then, we analyse high taxation, a policy that generates both transfers and a reduction in price by restricting demand while simultaneously creating an *(undesirable)* dead-weight loss. We show that the possible gains of high taxes are eventually outweighed by the dead-weight loss they generate, making MS a better policy through which to maximize welfare. MS achieves this price reduction more efficiently, as it avoids dead-weight losses within the luxury goods market.

We now consider only the "average" essential good with utility $\mathbb{E}[k_1 + k_2] \log x = k \log x$.[18] We first discuss the case of non-distortionary taxation to focus on the decline of transfers. Then, we allow distortionary taxation and show how the welfare gain leading to distortion is bounded as n approaches infinity (at any tax rate). However, the dead-weight loss is, in this case, independent of n.

A (relative) tax rate of τ implies that, if the consumer pays p, then producer receives $p(1-\tau)$. This tax is imposed on luxuries and distributed equally among all agents within the economy.

Distortion Free Taxes

Lemma 0.1. *The maximum distortion-free tax rate is $\tau = \frac{1}{2}$, and the amount raised by taxes is $W/2$ in terms of the numéraire.*[19] *Observe that this is independent of n.*

Proof. As luxury goods have linear utilities and their weights in the utility of the rich are ($\frac{1}{2}$ and $\frac{1}{4}$), then they will exchange all their endowment only if taxes are less than or equal to $\frac{1}{2}$. Both luxury goods are brought to the market, generating a revenue of $\frac{W}{2}$. $\square$

The revenue (R) is transferred anonymously to all agents. Under this regime, the demand for the essential good for the poor is $\frac{e+\frac{R}{(n+3)}}{p_{DT}}$ and for the pharma and the rich is

[18] This is a simplification which leads to the same results as the heterogeneity assumed under a uniform I.I.D. distribution. We deal with a much more general set of preferences in the main model of our paper (*see section*3).

[19] We say that a tax rate is distortion free if there is no distortion to equilibrium allocations of luxury goods

$\frac{k}{p_{DT}}$. Thus, the market clearing condition implies that:

$$1 = 3\frac{k}{p_{DT}} + n\frac{1 + \frac{R}{(n+3)}}{p_{DT}}$$

$$\implies p_{DT} = p^* \overbrace{+R - R\frac{3}{n+3}}^{\text{Price Effect}}.$$

The consumption of the poor is thus:

$$\frac{e + \frac{R}{(n+3)}}{p_{DT}} = x^* + \overbrace{\frac{R}{n}\frac{3k}{p^* + R - R/(n+3)}\frac{1}{p^*}}^{\text{Consumption Effect}}.$$

Welfare comparison

Now, we compare the welfare in the two regimes. We only need to analyse the welfare generated by consuming essential goods, as both MS and non-distortionary taxation allocate the non-essential goods efficiently. The consumption gain of the poor in MS is $\frac{2e(k-e)}{(n+2)e+k}\frac{1}{p^*} = \frac{2e(k-e)}{[p^*-2(k-e)]}\frac{1}{p^*}$, while the gain to the poor under transfers is $\frac{3kR}{(n+3)(p^*+R-R/(n+3))}\frac{1}{p^*}$. For transfers to outperform MS, it must be the case that:

$$\begin{aligned} \frac{3kR}{(n+3)(p^* + R - R/(n+3))}\frac{1}{p^*} &> \frac{2e(k-e)}{[p^* - 2(k-e)]}\frac{1}{p^*} \\ \implies \frac{1}{n+3}\frac{[p^* - 2(k-e)]}{[p^* + R - R/(n+3)]} &> \frac{2}{3}\frac{e}{R}\frac{(k-e)}{k}. \end{aligned}$$

For the above condition to hold, $\frac{e}{R}$ must be small, meaning that R must be large compared to e. When n increases and e and R are fixed, the inequality reverses, and segmentation is better than the alternative, as shown for the sample values selected in Figure 2.

When n is large, the welfare of the poor translates into overall welfare. We show in Appendix section A.2 that as $n \to \infty$, $W_{MS} > W_{DT}$, for all $k > 1$.

Distortionary Taxes

If the tax rate is above half, then the demand function of the rich changes. They will either only use universal goods to buy essential goods, or else sell their luxuries to buy essential goods at a higher price. The resulting kinked demand function can be seen in Figure 3a. The reader should observe that there is a large degree of distortion, $\frac{W}{2}$, because no luxuries are traded for each other. The equilibrium price of luxury goods must be 1/2. If the rich use luxuries to buy essential goods, then the relative price they face is $\frac{p}{2(1-\tau)}$. Thus, their demand can be given by:

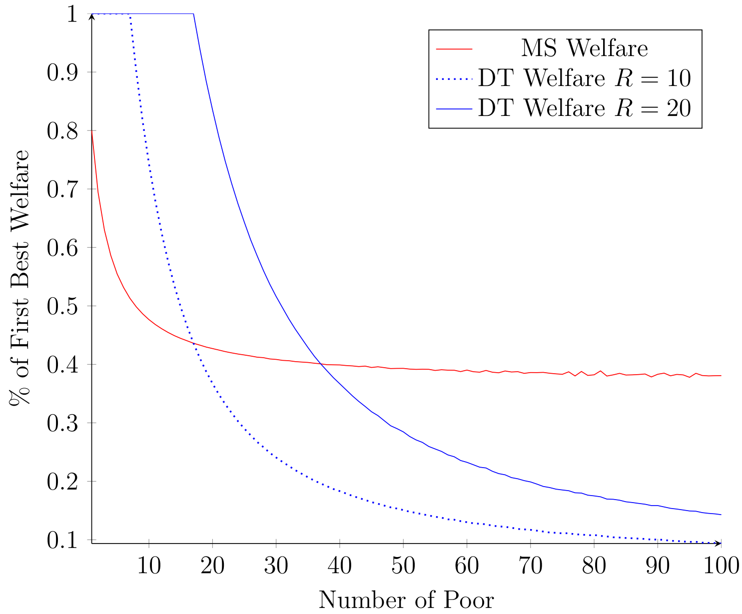
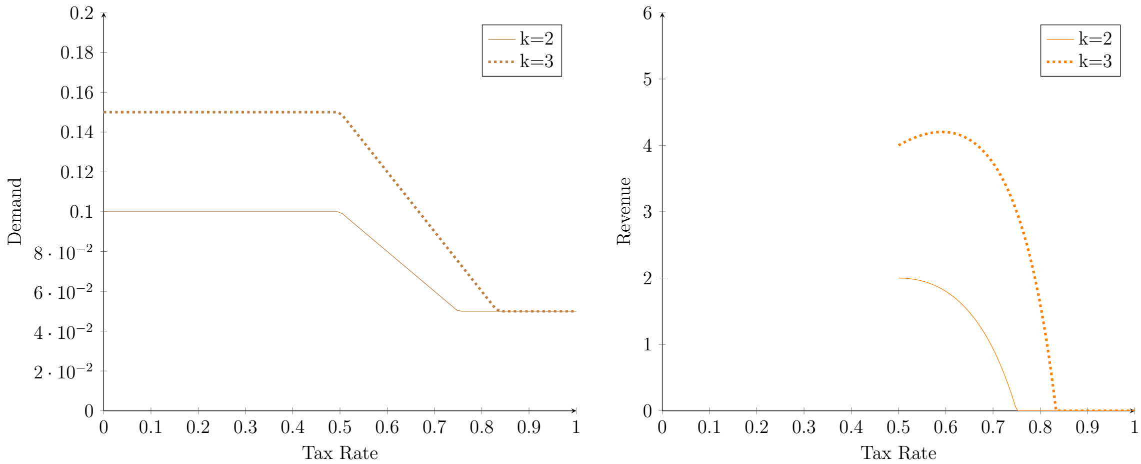


Figure 2: MS vs DT, with $e = 1, k = 2$

$$x_1 = max\{2(1-\tau)\frac{k}{p}, \frac{1}{p}\} \tag{1}$$



(a) Rich Demand

(b) Planner Revenue

To compute the total tax revenue collected by the government, we first determine the income from luxuries that the rich must earn to fulfil their demand for essentials. Since their total demand is given by equation 1, they must earn $2(1-k)\tau - 1$. Let A denote the amount they need to sell to receive this much income. As the price of the luxury is $\frac{1}{2}$, the following must hold:

$$A \times \frac{1}{2}(1-\tau) = 2(1-k)\tau - 1$$

Let the revenue raised by the planner be R, where,

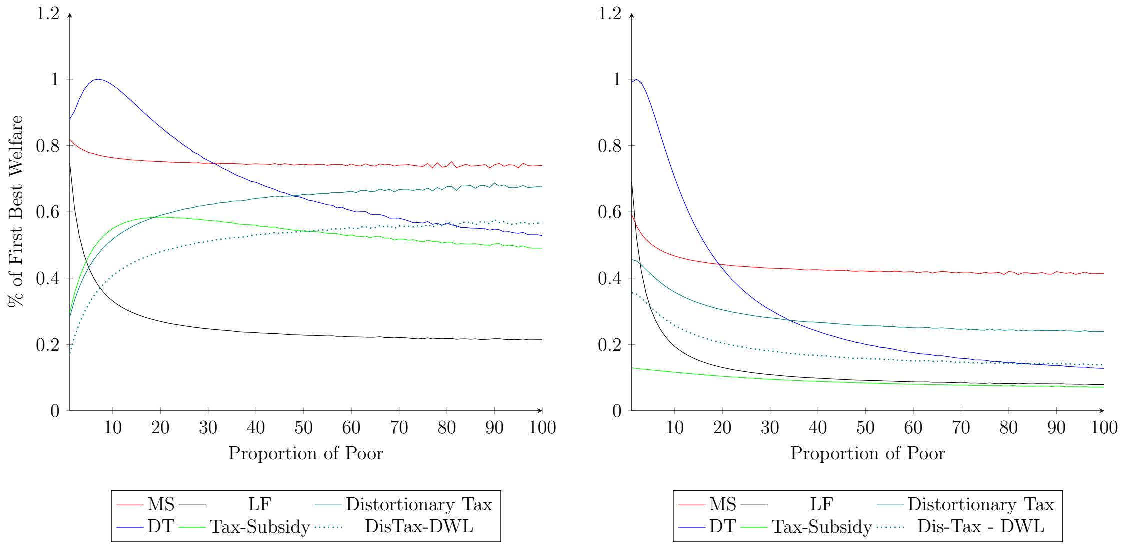
$$R = 2 \times A \times \frac{1}{2}\tau = 4k\tau - \frac{2\tau}{1-\tau}. \tag{2}$$

Note that the above equation demonstrates that government revenue is independent of the endowment of the luxury good. The tax rate cannot exceed 1, implying that the tax revenue is bounded. Thus, we can see that the gain from distortion is small. On the other hand, the dead-weight loss can be arbitrarily large. Further, any price effect from distortionary taxes is lower than complete segmentation. Putting the above facts together explains our result. For formal proof, see Appendix section A.2.

We now provide graphical representations of numerical computations of social welfare under different regimes, namely MS, taxation with a subsidy on essential good (TS), and commodity taxation with direct transfers (DT), comparing their performances.

Numerical Experiments

We now plot the total welfare achieved by different regimes in our construct with the luxury good endowment (I) set as $W = 20$ and $k = 2, 3$ (*see Figures* 4 and 5).



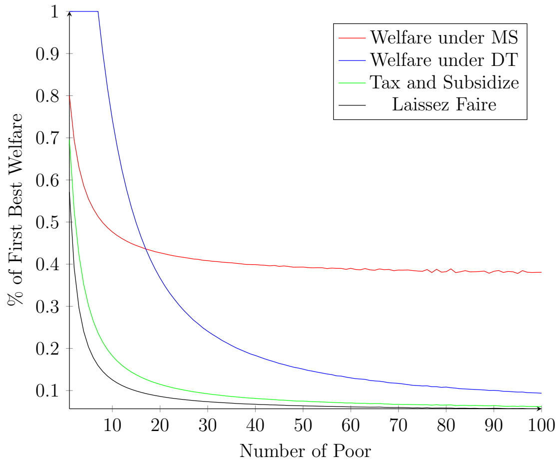
(a) $k = 2, W = 20$ (b) $k = 3, W = 20$

Figure 4: Welfare of the Society

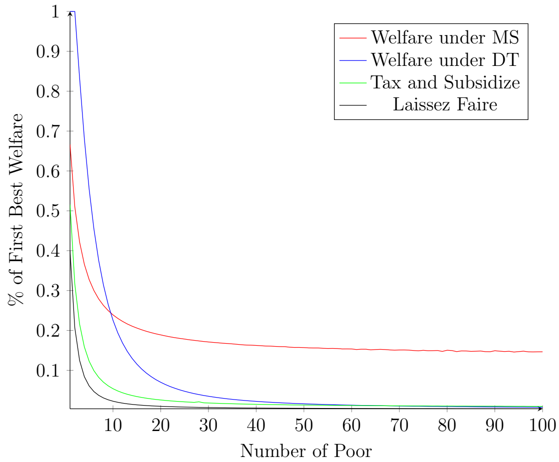
3 Model

Goods and Agents

A bundle of goods is denoted by a k-dimensional vector $X = \{x_1, ...x_k\}$ chosen from a compact subset $\mathcal{G}$ of the k dimensional reals, $\mathbb{R}^k$, $X \in \mathcal{G} \subset \mathbb{R}^k$. These are then divided



(a) $k = 3$, $W = 20$



(b) $k = 3$, $W = 20$

Figure 5: Welfare of the Poor

into 2 distinct sub-classes which we term **essential** and **non-essential**. The economy has only a finite number of agents. We refer to each agent by ω and the (finite) set of agents by Ω, where $|\Omega| = H$. These agents are heterogeneous with respect to both endowments and preferences. Preferences are separable over goods, such that for individual ω:

$$U^{\omega}(X) = \sum_{k=1}^{K} b_k^{\omega} u_k(x_k)$$

where b_{ω} is an idiosyncratic component and $u_k(x_k)$ varies good by good, yet is constant across individuals. Every individual is born with an endowment $e^{\omega} \in \mathcal{G}$. Essential goods and non-essential goods are defined as follows:

Essential goods: A good x_k is termed essential if $u'(0) = \infty$.
Non-essential goods: A good x_k is termed non-essential if $u'(t) = c > 0 \quad \forall t$.

There is one non-essential good which is universally endowed and thus we call it the universal good. With respect to endowments, individuals are divided into (at least) 3 subgroups, specifically:

Poor: An agent ω is deemed 'poor' if they are *only* endowed with the universal good. We think of this good as manual labor or time. They are collectively Ω_p, where $|\Omega_p| = n$.
Rich: An agent is deemed 'rich' if they are endowed with a positive amount of any luxury good *(along with the universal good)*. They are denoted Ω_r, where $|\Omega_r| = m$.
Don: An individual is deemed a 'don' if they are endowed with a positive amount of any essential good *(along with the universal good)*. They are denoted Ω_d where $|\Omega_d| = d$.

We assume that Dons and Rich are disjoint groups. Thus, $H = n + m + d$. This lets us completely define our environment as an economy $\mathcal{E}$, in which:

$$\mathcal{E} = (\mathcal{G}, \Omega, \bigcup_{\omega \in \Omega} \{U^\omega\}, \bigcup_{\omega \in \Omega} \{e^\omega\}),$$

a tuple of goods, individuals, utilities and endowments.

The Social Planner

The social planner is utilitarian, wishing to maximize the sum of utilities of the agents within the economy. They can levy taxes on goods traded in the market. As such, we rule out taxing endowments directly and allow taxes only on traded endowments. However, the planner is constrained to budget balance (*i.e.,* they can only commit to revenue-neutral policies). Hence, they face a trade-off between creating equality in the consumption of essential goods on the one hand and causing excess distortions by taxing non-essential goods on the other.

Policies Available to the Social Planner

The planner can define a function which maps each **traded endowment** to a **budget set**, and the policy space is simply a family of such functions. Thus a planner chooses suitable policy parameters (per the policies defined below), mapping each traded endowment to a final budget set. We analyze four policies which are available to the planner. The definition of each policy is as given below:

Taxation and Direct Transfers (DT or transfers henceforth): The planner can tax trades on non-essentials and use the tax to provide direct transfers to agents. Formally, DT policy is defined for each traded endowment level $(x - e)$, selecting values of τ_k and R, such that:

$$\mathcal{B}_{DT}(x - e) = \sum_{k=1}^{K} p_k(1 + \tau_k \mathbf{I}_{x_k - e_k})(x_k - e_k) + R \leq 0,$$

where $\mathbf{I}_{x_k - e_k} = 1$ whenever $x_k - e_k \geq 0$, R is the anonymous transfer received by each individual in the economy, and $\tau_k \geq 0$ are the taxes set on each good k.

Taxation and Subsidy (TS or subsidies henceforth): The planner can tax trades on non-essentials and use the tax to subsidize the essential goods to agents within the economy. Formally, the TS policy is defined such that:

$$\mathcal{B}_{TS}(x - e) = \sum_{k=1}^{K} p_k(1 + (\tau_k - \sigma_k)\mathbf{I}_{x_k - e_k})(x_k - e_k) \leq 0,$$

where $\mathbf{I}_{x_k - e_k} = 1$ whenever $x_k - e_k \geq 0$, $\tau_k \geq 0$, and $\sigma_k \geq 0$ are the taxes and subsidies on each good, k, respectively.

Note that we assume that all taxes are paid by net sellers in DT and TS.[20]

Quantity Rationing (QR or rationing henceforth): For any essential good x_k, the planner can decide the quantity level q_k, such that each individual can buy the given quantity of a good at a rationed (lower) price $\hat{p}_k$. Formally, the QR policy is defined as:

$$\mathcal{B}_{QR}(x-e) = \left\{ \sum_{k=1}^{K} p_k(x_k - e_k) \leq 0 \right\} \bigcap \{x_k - e_k \leq q_k\}_{k=1}^{K}.$$

Market Segmentation (MS or segmentation henceforth): Finally, we introduce a new policy:- market segmentation, where an individual incurs a tax if they trade one type of goods for another. In our setup, the policy entails segmentation between a set including the universal good and essential goods, and another set including all other non-essential goods.[21] Formally, MS policy is defined as picking τ_{MS} and R_{MS} for each endowment level (e), such that:

$$\mathcal{B}_{MS}(x-e) = \left\{ \sum_{\text{essential + Universal}} p_k(x_k - e_k) \leq y + R_{MS} \right\} \bigcap \left\{ \sum_{\text{luxuries}} p_k(x_k - e_k) \leq -(1 + \mathbf{I}_y \tau_{MS})y \right\},$$

where y is the income accrued from selling non-essential goods spent on essential goods, and $\mathbf{I}_y = 1$ when $y > 0$. $\tau_{MS} \geq 0$ is the segmentary tax level designated, while R_{MS} is the anonymous transfer each individual receives from the revenue generated from segmentary taxes. We refer to $\tau_{MS} = \infty$ as complete market segmentation. At this level, $R_{MS} = 0$ and $y = 0$.

Utility Maximization

We assume that individual agents are price takers and maximize their utility subject to a budget constraint and the actions of the social planner. They take prices (possibly non-linear), endowments and social planner's policy as given. Their problem is:

$$\max_{X \in \mathcal{B}(x^\omega - e^\omega)} U^\omega(X),$$

where $\mathcal{B}(x^\omega - e^\omega)$ represents the budget set corresponding to the endowment of the

[20] Of course, this in inconsequential, as basic public finance theory reveals that who pays the tax is inconsequential to who bears 'the true burden' of the tax. The incidence of the tax thus depends on the price elasticity of the buyer and sellers (see Chapter 6 of Mankiw (2011).

[21] We take the term universal good to be a non-essential good in keeping with the example discussed in section 2.

agent and the planner's policy.

We will always remain in an environment where the poor exist on the boundary (*i.e.,* they only consume essential goods in a *laissez-faire* economy. Formally this would imply that $u'(\frac{m}{p}) > b_k^\omega \quad \forall \omega, k$, where m is the income of the poor, and p is the price of essential goods in such a *laissez-faire* setup.

Though we are considering individual heterogeneity in preferences with respect to the essential goods, we do not consider any one essential good to be more important than the other on aggregate. We now state an assumption which captures this notion of equal importance, which we term aggregate symmetry.

Aggregate Symmetry (Definition): Let us consider an economy with K essential goods. The preferences for each individual are defined by a vector of parameters $b = (b_1, ...b_k)$. We say that an economy satisfies aggregate symmetry if the following is true: Suppose that the preferences are distributed according to $\mu \in \Delta b$ and the CDF corresponding to μ is $F(b_1, ...b_n)$, then for any $i, j \leq n$ it must hold that:

$$F(..., b_i, .., b_j) = F(..., b_j, .., b_i...)$$

.

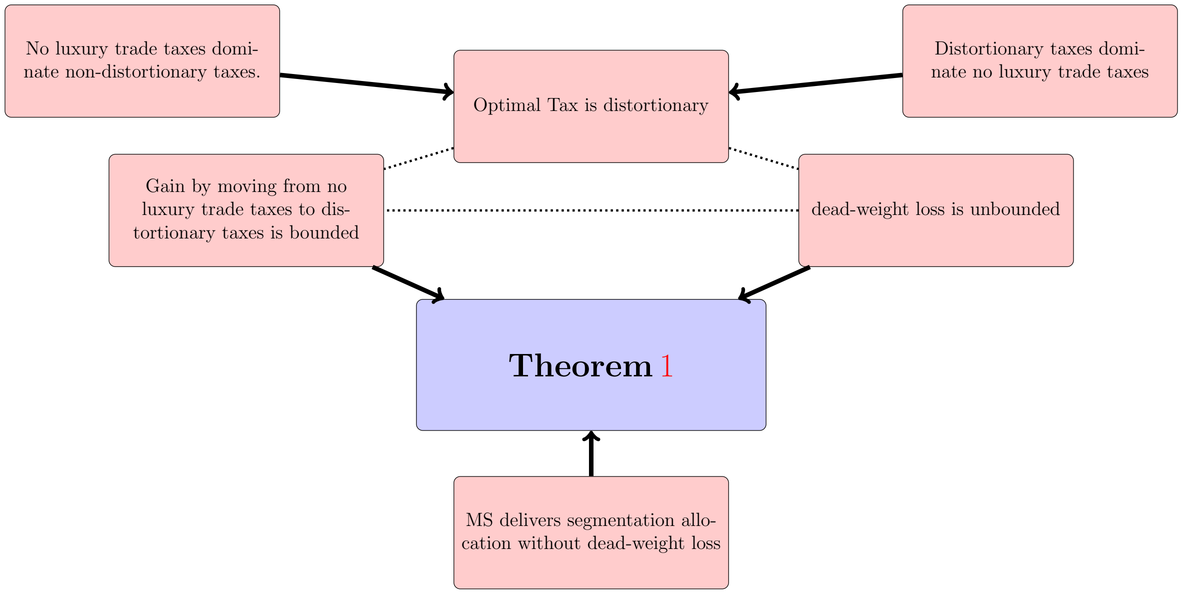
We will see that one implication of this assumption is that the relative benefits of different policy choices of the planner are independent of the number of goods. Hence, we can prove general results going forward by only analysing the case with one (or two) essential good(s).

Lemma 0.2. *Under aggregate symmetry, the social planner treats essential goods symmetrically.*

Proof Idea: The planner wants to maximize social welfare by using policy parameters as defined above. First, note that the frontier of allocations encloses a convex set because demand (excess supply) functions are convex in terms of revenue spent (as demand is price-inelastic). Now suppose the planner chooses an asymmetric final allocation. However, a symmetric permutation of the allocation remains feasible because of aggregate symmetry. Due to the convexity of individual utilities, the intermediate allocation has a strictly higher value to the planner[22], which is an apparent contradiction. Thus, goods must have symmetric allocations. For formal proof see B.

Lemma 0.3. *As all essential goods are treated symmetrically by the planner, it is sufficient to consider a case in which we have one (or two) essential good(s).*

[22] As sums of convex functions remain convex.



Proof Idea: As all goods are treated symmetrically by the planner, we can simply constrain the planner to consider those policies in which all goods are treated identically. This reduces to a problem where the planner only chooses a policy for a solitary good, and the rest of the goods follow the same policy owing to symmetry. For a formal proof see B.

Given that it is sufficient to consider one essential good, let us call it x with price p. There is one unit of essential good owned by one don. We now state our main assumption:

Assumption 1. Essential goods are price-inelastic.

$$\frac{\partial}{\partial p_e}\left[p_e x_e(p_e, ..)\right] \gneq 0,$$

where p_e is the price of the essential good, and x_e is its respective demand.

Remark. As discussed before, n denotes the number of poor agents within the economy.

Our first theorem compares complete market segmentation (MS) with taxation with direct transfers (DTs).

Theorem 1. *Under assumption 1 and given significant gains from trade within luxury goods, $\exists N$ such that for all $n \geq N$, MS dominates DT.*[23]

We prove this theorem through a series of claims. We first demonstrate that, if the essential good is price inelastic and the number of poor in the economy is high,

[23]Significant gains from trade means that the dead-weight loss within the economy at the tax level of no trade between luxury goods is greater than G, where: $G = n(U(\frac{e+\frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + m(U(x(p*,\tau)) - U(p_{DTS},\tau)) +$
$U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}$, we show this to be bounded.

then commodity taxation restricting all trade in luxuries gives greater social welfare than for any level of non-distortionary taxation.[24] The reason for this is three-fold. First, if n is high, then the effective transfer to the poor is low. Secondly, with the inelasticity of essential goods, non-distortionary taxation does not decrease the demand of the rich substantially and consequently merely increases prices. Third, the gain from non-distortion is bounded and independent of n. We now formally state the claim.

Claim 1.1. *If assumption 1 holds, $\exists N \in \mathbb{N}$, such that, for $n \geq N$ social welfare in a regime where commodity taxation is high enough to restrict all trade in luxury goods dominates welfare for any non-distortionary tax regime.*

Proof. For formal proof, see Appendix B. □

The second step is to show that a taxation level which leads to no trade in luxuries – we call this the 'no luxury trade' tax level (τ_{nlt}) – is dominated by some distortionary level of taxation which permits some trade. This again arises due to inelasticity. Due to the price inelasticity of the essential good, the tax revenue increases with tax rate but suddenly becomes zero at the segmentation level. We show that the gain from going to the segmentation level is always dominated by the revenue that can be generated by taxing a little less. We now formally state this claim.

Claim 1.2. *Under assumption 1, there exists a distortionary level of taxation τ^*_{DT} that achieves a higher welfare than τ_{nlt}.*

Proof Idea. The key observation is that, while demand behaviour varies continuously at the cutoff tax τ_{nlt}, the revenue from said tax falls to zero discontinuously. For a formal proof, again see Appendix B.

Let $\tau_{DT}*$ be the optimal tax, and let the revenue it generates be $R_{DT}*$. We now show that the welfare gain from moving from τ_{nlt} to $\tau_{DT}*$ is bounded and is thus independent of n.

Claim 1.3. *Let the welfare gain generated by revenue R, where the relative number of poor is n, be $G_n(R)$, then:*

$$\exists \quad G^*(R) \quad \textit{such that} \quad \forall n \quad G_n(R) \leq G^*(R)$$

Proof Idea. Note that taxes can only be levied on non-essential goods and thus the revenue generated is independent of n. Although some revenue (R_{DT}) is generated and distributed to all agents at τ^*_{DT}, it does not help the poor greatly, as per capita transfers decrease with n. Moreover, under price inelasticity, these transfers translate into price increases and the welfare gain is limited (see Appendix B for a formal proof).

[24] High commodity taxation that leads to restricting all trade in luxuries is termed a *no luxury trade tax.*

Claim 1.4. *If the dead-weight loss arising due to optimal distortionary taxation is more than the above gain (G^*), then MS outperforms segmentary taxation.*

Proof. Welfare under Market Segmentation (W_{MS}) is given by:

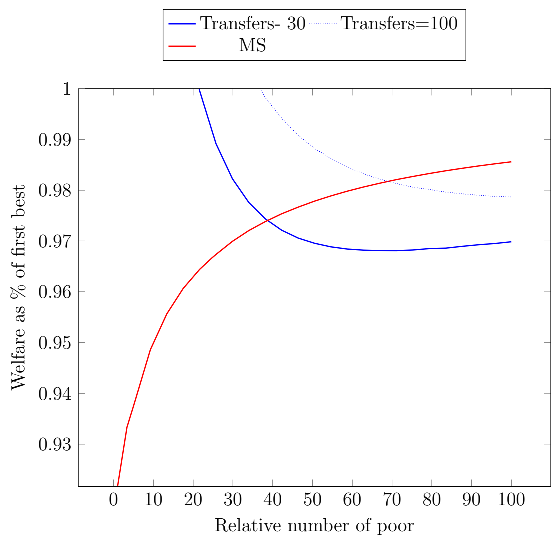
$$W_{MS} = W_{NLT} + DWL$$
$$W^*_{DT} = W_{NLT} + G^*$$

and hence,

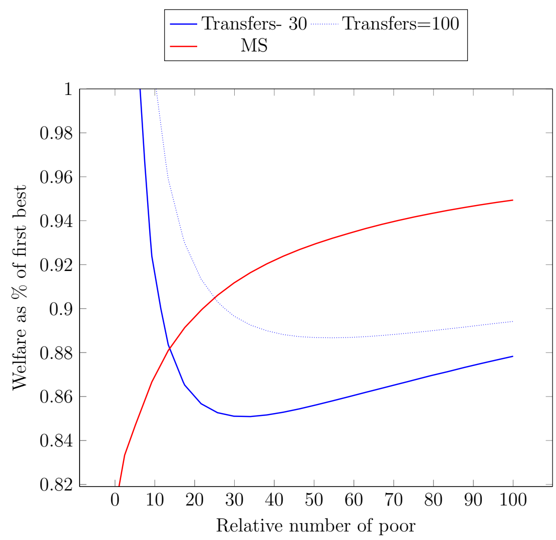
$$W_{MS} - W_{DT*} > 0 \iff DLW > G^*$$

$\square$

Now we provide some computational experiments in Figure 6 to show that, under conditions of price inelasticity in essential goods, welfare under MS swiftly overtakes welfare under DT.



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$



(b) Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

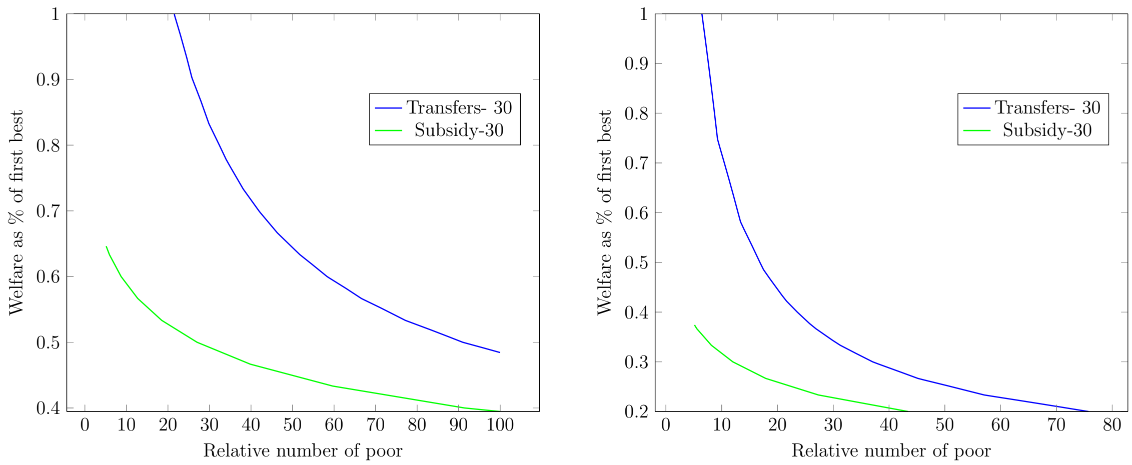
Figure 6: MS vs DT

Having proven that MS dominates DT when n is large and essential goods are price inelastic, we now show that DT dominates a subsidy regime under the same conditions.

Theorem 2. *Under assumption 1, given any revenue R, $\exists N \in \mathbb{N}$, such that, $\forall \quad n \geq N$, direct transfers are more efficient than a subsidy on essential goods.*

Proof Idea. Fixing the amount of tax revenue makes the dead-weight loss across regimes equal. Thus, we only need to focus on the distribution of essential goods within the economy. Note that subsidies work by effectively driving a price wedge between sellers and buyers. This, in turn, increases the commodity supply by increasing the price sellers receive. On the other hand, transfers increase the price for **both** the rich and sellers, thus decreasing their consumption and increasing the net supply to the poor.

This means that, even though the price increases may be small within the direct transfers regime relative to the subsidy case, the inelasticity of demand implies that *a large change in price for a few will not be as effective as a small change in price for many.* The number of people affected by direct transfers is always larger than for subsidies and thus they are more efficient. For a formal proof see Appendix B. Corresponding computational experiments are provided in Figure 7.



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$ (b) Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

Figure 7: Transfers vs Subsidies

Theorems 1 and 2 together present the primary insight offered by this paper. If the essential good is inelastic and society is highly unequal in terms of endowment distribution, then MS outperforms taxation-based instruments, specifically transfers and subsidies.

However, one may argue that outcomes generated by MS can also be implemented using direct quantity rationing (QR). We now show that, while QR mimics MS if there is just one good, the poor are better off under MS if there is more than one essential good and heterogeneity in terms of preferences over essential goods.[25]

Theorem 3. *Let us consider an economy with two essential goods over which preferences are distributed as defined under aggregate symmetry. MS improves the welfare of the poor to a greater extent than the quantity rationing of essential goods.*

Proof Idea. MS takes advantage of the heterogeneity in preferences over essential goods which QR cannot do. Under MS, the rich consume essential goods just as the poor do. However, under QR, an upper limit is set on the consumption of **each** essential good. Thus, the rich can consume as much as they can of each essential good. This implies that

[25]It is worth noting that quantity rationing can be implemented by a non-linear tax of infinity over a certain level of consumption of the essential good (see Gadenne (2020)).

less is left for the poor under QR, particularly those who vastly prefer one essential good to another. As Weitzman (1977) notes, price systems are better positioned to address the heterogeneity of needs arising within a population. MS dominating QR is a perfect example of this astute observation. Again, for a formal proof see Appendix B.

Thus we have shown that complete MS (*i.e.,* not allowing **any** trade between essential and luxury goods), can serve as a more effective policy than direct transfers, subsidies and quantity rationing, under certain conditions. However, do we necessarily want 'completely' essential and luxury goods markets? In particular, a planner might allow what we call a segmentary tax to be raised. This tax is levied if the income generated by selling luxury goods is used to buy essential goods, though not if luxury goods are traded amongst themselves. We call this policy 'partial MS'.

The next theorem demonstrates that partial MS is always better than complete MS in our construct, as at least some trade should be allowed between luxury goods and essential goods. This welfare gain occurs as the revenue earned by lowering the tax a little from the segmentary level improves welfare more than the decrease in welfare arising due to an increase in the price of essential goods.

Theorem 4. *Under assumption 1, partial MS dominates total MS.*

Proof Idea. The intuition for this theorem is essentially the same as that of claim 1.2 of Theorem 1. Given the price inelasticity of essential goods, the tax revenue increases in tax rate but becomes suddenly zero at the segmentation level. The gain of complete segmentation is always dominated by the revenue generated by taxing a little less as the demand behaviour varies continuously at the cutoff tax τ_{cms}. In contrast, the revenue from such a tax falls to zero discontinuously (*please refer to Appendix B for the proof*).

As elasticity plays a key role in the above argument, we provide Figure 8 which compares consumption in the partial segmentation regime to the complete segmentation regime with varying levels of elasticity. As the price elasticity of essential goods increases, the difference between the two regimes decreases because the revenue loss due to tax decreases with the increase in elasticity. Nevertheless, for all levels of price elasticity less than 1, it is clear that partial segmentation dominates complete segmentation.

As complete MS is never optimal in our construct, we now characterize the optimal segmentary tax. Moreover, notice under those conditions where Theorem 2 holds (*i.e.* large n and price inelasticity of essential goods), we know that the revenues these taxes generate should be used as direct transfers rather than subsidies.

Theorem 5. *At the optimal segmentary tax (τ_s),*

$$\pi\lambda_r\left[\frac{\partial R_{MS}}{\partial\tau}(1+\tau)-\Big(p_1(x_1-e_1)+p_2(x_u-e_u)-R_{MS}\Big)\right]+\pi_d\lambda_d\frac{\partial R_{MS}}{\partial\tau}+\pi_p\lambda_p\frac{\partial R_{MS}}{\partial\tau}=\sum\mu_k\sum\frac{dx(p,\tau)}{d\tau}$$

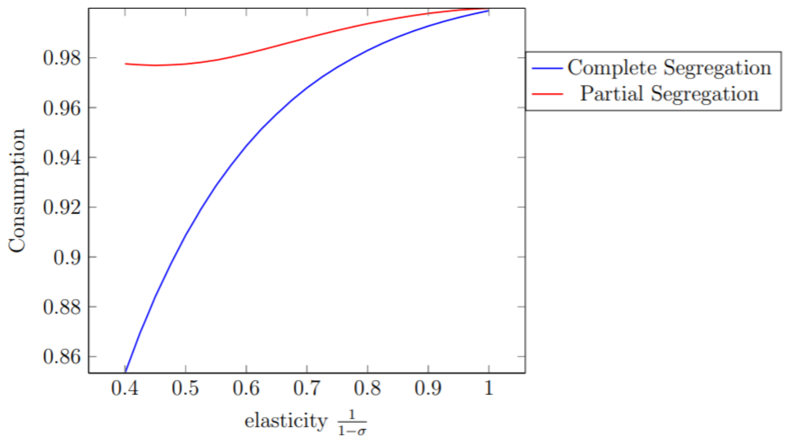


Figure 8: Performance vs Elasticity

or

$$\frac{\partial R}{\partial \tau_s}\left[\lambda_r(1+\tau)\pi_r+\lambda\pi_d+\pi_p\lambda_p\right]-\sum \mu_k \sum \frac{dx(p,\tau_s)}{d\tau_s}=\pi_r\lambda_r\left[\left(p_e(x_e-e)-(x_u-e_u)-R_{MS}\right)\right]$$

where π_r, π_d, π_p are the weights of the rich, the don and the poor, respectively, x_e denotes the essential good and x_u the universal good. R_{MS} is the per capita transfer, λ is the marginal utility of income for each group, and μ is the multiplier associated with the resource constraint.

Proof Idea. To understand this condition, let us consider the cost and benefits of using a segmentary tax. Increasing the segmentary tax has two major effects. First, it increases the 'segmentary tax' revenue which can be used as transfers (R_{ms}). Second, it decreases the price of essential goods by making them more expensive for the rich, thus effectively 'freeing' up the good for the poor. In the second formulation of the above equation, the left-hand side represents the weighted utility to society of the transfer which an additional tax unit allows the planner to provide and also the increase in welfare by 'freeing up' the essential good. As τ_s increases, so do the transfers ($\frac{\partial R}{\partial \tau_s} > 0$). Moreover, the essential good is freed up ($\frac{dx(p,\tau_s)}{d\tau_s} < 0$) which increases utility. The right-hand side represents the loss to the rich. The loss of the rich is captured by the fall in x_e. Notice that if the rich have a low weight π_r in the equation delineated above, then the optimal segmentary taxes are high and approach complete segmentation levels. For a formal proof see Appendix B.

In this section, we presented a formal general equilibrium model to demonstrate the conditions under which the policy of market segmentation dominates direct transfers, subsidies and direct quantity rationing. We also discussed how partial MS is always better than complete MS, where the essential good is price inelastic and then went on to characterize the optimal segmentary tax. In the next section, we will discuss some potential applications of these taxes and how they can be implemented.

4 Applications and Implementation of Segmentary Taxes

This section discusses how segmentation and segmentary taxes potentially have important applications in dealing with issues facing today's world. We also discuss how they can be implemented and how they are related to other policy instruments.

4.1 Applications

4.1.1 Automation

An issue that will starkly affect the economy and distribution in the near future is automation. Scott Santens summarises the problem succinctly in a Boston Globe 2016 article, "nothing humans do as a job is uniquely safe anymore. From hamburgers to healthcare, machines can be created to successfully perform such tasks with no need or less need for humans, and at lower costs than humans." He is not the only one to have raised this as a matter of concern. Recent work by prominent economists highlights the need to deal with automation-driven inequality [*see* (Aghion et al., 2017); (Acemoglu and Restrepo, 2019); (Mookherjee and Ray, 2020)].

In this section, we study the application of the various policy instruments discussed in the preceding section to deal with the problems that may arise due to automation. We argue that automation can have two different types of impacts on the endowment/skill distribution in the economy. First, it can lead to a situation in which machines start performing many tasks, consigning erstwhile 'skilled' labor to unskilled work. We call this the 'displacement effect' of automation. It thus increases the relative number of unskilled (poor) workers in society. On the other hand, automation can also have a different impact on the endowment distribution if robots erode the value of unskilled workers in the labor market without having a considerable impact on skilled jobs. We call this the 'erosion effect'.

We find that this distinction is crucial because it leads to different policy prescriptions. Since the displacement effect of automation renders skilled workers unskilled, this is akin to n (*i.e.* the relative number of poor) increasing in our model. As discussed in Theorems 1 and 2 of this paper, if essential goods are price inelastic and there are sufficient gains from trade within the luxury good market then, as n increases, so MS dominates DT and the subsidy regime. Hence, we argue that if automation leads to high displacement within the economy, then MS can potentially be more effective than other policies at dealing with its distributional impact.

However, if automation produces more of the erosion effect, then direct transfers are more effective at dealing with the issue than segmentation. Realize that, when the value of the universal good in the economy decreases, the purchasing power of the poor decreases rapidly. Thus increasing the purchasing power of the poor through direct transfers becomes very important, causing the DT regime to dominate the MS and TS regimes which only affect welfare by lowering prices of essential goods.

To study the impact of the 'erosion effect' more formally, we go back to our basic model in Section 3. We model this effect as a fall in the value of parameter b associated with the universal good (e_3) in the utility function of all consumers in the economy. We next state a theorem formally proving that, as the value of the universal endowment falls in the utility functions of individuals within an economy, so DT dominates MS.

Theorem 6. *As the weight of the universal good in the utility function falls (i.e. $b \to 0$), direct transfers are more effective in improving social welfare than MS.*

We again prove this theorem in a series of claims. The first claim establishes a strong intuitive and mathematical link between the weight of the universal good (b) and the actual endowment of the universal good for each individual within an economy.

Claim 6.1. *Let b be the weight associated with the universal good in individuals' utility functions, and let e be the (common) endowment of the universal good. As $b \to 0$, the equilibrium allocation produces the same welfare as $e \to 0$.*

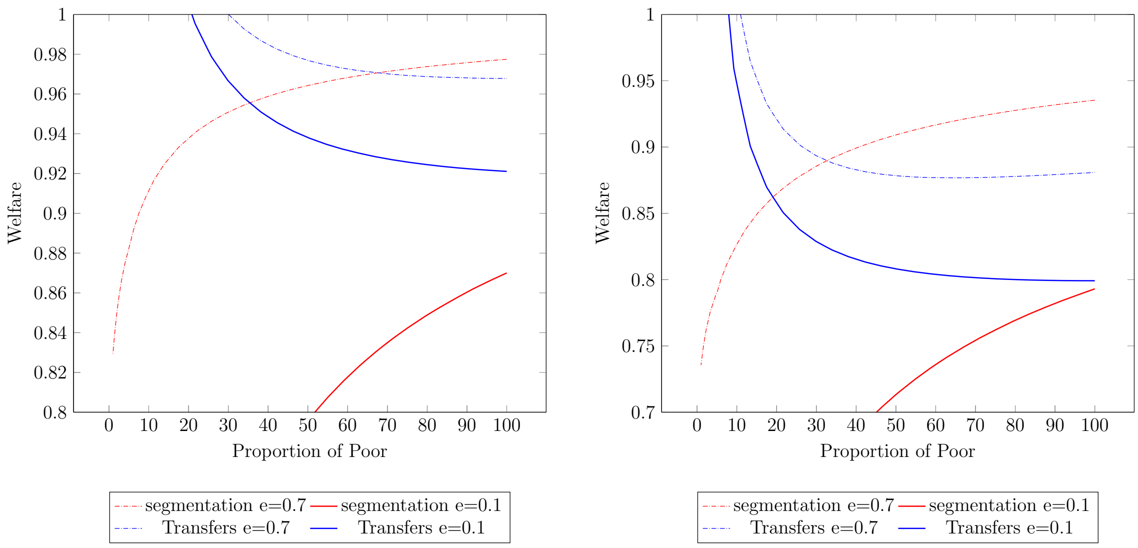
Proof Idea. The intuition behind this claim is simple. If the marginal value of the good goes to zero, then nobody is willing to pay for it in the market. Consequently, the income it generates is zero. This is akin to an individual losing his endowment as it ceases to be valuable. The formal proof is given in Appendix B.

This claim implies that we can study a decrease in the marginal value of the endowment as a decrease in the amount of endowment holding the marginal value constant. Thus, we now discuss various groups' relative marginal utilities as the universal good's endowment goes to zero.

Claim 6.2. *As the endowment of the universal good goes to zero, the ratio of the marginal utilities of the poor to the rich goes to infinity. Thus, only the poor remain welfare relevant.*

Proof Idea. The intuition for this claim is straightforward. We know that the poor are solely endowed with the universal good, e. If that falls, they cannot even consume a little of the essential good. We have assumed that, as consumption of the essential good goes to zero, $u'(x) \to \infty$. As the rich (and the dons) have positive incomes from selling other goods with which they are endowed, so their consumption does not fall below a certain level. Thus, their marginal utility is bounded away from 0. Taken together, this proves the claim.

Finally, the claim asserted below completes the proof.



(a) Price Elasticity $|\frac{1}{1-\sigma}| = 0.8$

Figure 9: Price Elasticity $|\frac{1}{1-\sigma}| = 0.6$

Claim 6.3. *Considering the welfare of the poor, as the endowment of the universal good in the economy goes to zero, direct transfers dominate complete market segmentation.*

Proof Idea. We know that the MS regime improves social welfare by lowering the demand of the rich and thus, lowering the prices of essential goods, making it easier for the poor to buy more. However, if poor people have no income because they have no endowment, even lower prices cannot improve their welfare. Thus increasing the purchasing power of the poor through direct transfers becomes very important if automation erodes even the minimal purchasing power of even a few people within the economy. For a formal proof please refer to Appendix B.

Theorem 7. *As the value of the universal endowment falls in the utility functions of individuals in an economy ($b \to 0$), direct transfers are more effective in improving social welfare than a subsidy on the essential good.*

Proof Idea. This theorem follows using similar claims to those used above.[26] As the weight associated with the universal good goes to 0 ($b \to 0$), it is as if the endowment of universal good (e) is going to zero. If e goes to 0, then the ratio of marginal utility of the poor to the rich (or the don) goes to infinity. Moreover, as in the case of MS, subsidies also work by lowering prices of the essential goods. However, as $e \to 0$, the poor have little to no purchasing power. Hence, increasing the purchasing power of the poor through direct transfers becomes very important, giving us the result. Again, the formal proof may be found in Appendix B.

[26] In particular, claims 6.1 and 6.2 remain exactly the same. Claim 6.3 needs to be modified to now discuss DT vs TS (rather than DT vs MS). However, the basic idea remains the same. For a formal proof see Appendix.

4.1.2 Dealing with such trends as the 'superstar phenomenon'

Recent work shows that emergent trends in the 21st century, such as the 'superstar phenomenon' (Scheuer and Werning, 2017) and markets with winner-take-all characteristics (Rothchild and Scheuer, 2016) render conventional policies like non-linear labor taxation only mildly effective. Scheuer and Slemrod (2019) note and discuss various phenomena as to how taxation of the super-rich has become increasingly challenging in recent years. In such labor markets, 'superstars' or 'winners' cannot optimally face a high marginal income tax on labor without creating substantial dead-weight loss within the economy given that there are convex returns to the effort exerted. We argue here that MS can help policymakers to deal with these challenges.

In Appendix C, we introduce production and allow non-linear labor taxation. We show that our fundamental insight – *i.e.* that MS can protect the prices of essential goods from rising to unaffordable levels without very large dead-weight losses – still holds in an economy with production, heterogeneous labor productivity, and non-linear taxation (Saez, 2004). The intuition for the results otherwise essentially remains similar. If the essential goods are inelastic, then dead-weight losses in terms of production due to the high non-linear labor taxation required for equity concerns is large. Again, it is better to segment consumption into essential and luxury goods under this scenario. The planner can then cap expenditure on essential goods, leading to a more equitable distribution of essential goods while avoiding dead-weight loss in production.

In subsection C.4.2 we show that, in our construct, if there is significant mass on the upper tails of the skill distribution (which can be interpreted as the presence of superstars within the labor market), the dead-weight loss increases under non-linear taxation with transfers, yet is unaffected under MS. Hence, the gains from using MS over non-linear taxation increase when 'superstars' are present within the labor market.[27]

When the planner uses MS, agents still have incentives to work and produce because they can use their incomes to consume luxuries. However, they cannot spend their 'excess superstar income' on essentials, which keeps their prices low. Thus our policy allows the planner to uncouple the prices of essential goods from incentives to produce. To put this in terms of an example, even if healthcare and affordable housing are rationed through MS, people still have incentives to work hard to consume more i-phones, Nike sneakers, trips to Europe and so forth. Thus, when labor markets have dense right-tailed skill distributions, such segmentation can avoid inefficient levels of taxation while maintaining a more equitable distribution of essential goods.

In general, it should be noted that any phenomenon which increases inequality in terms of endowment distribution while increasing dead-weight loss in relation to taxation,

[27] Note that we demonstrate our results for a discrete economy, thus making no assumptions at all about the return to ability being convex or concave. Indeed if these are convex our results would be starker.

renders MS a more effective policy than either transfers or subsidies in our construct. We believe that this general insight of our paper might also be applicable in other salient socioeconomic spheres.

4.1.3 When should there be a price to cut the queue?

As a final discussion on the real-world applications of our findings, we note that our discussion comparing the effectiveness of MS to commodity taxation (DT or TS) can also be thought of as comparing the effectiveness of QR to DT or TS. QR is a traditional policy followed in many economies such as the UK's National Health Service (NHS).[28]

Our model can be engaged to provide a more informed discussion on whether the NHS should continue to operate as it does or whether cash transfers with a privatised health care system would fare better. Our model suggests that if healthcare provision is inelastic in terms of its demand and supply and there is significant income inequality within the population, providing health services through rationing would be better solution.

Perhaps, an application of Theorem 4 of our model suggests specific changes that can be made to the rationing system. Theorem 4 proves that, if essential goods are price inelastic, it is better to segment the essential good market partially, implying that under our assumptions, even in such NHS-style systems which practically ration essential goods, **there should always be a price to cut the queue.** The revenue raised should be used in the form of transfers to improve the distribution of health services in the economy. This result resembles Dworczak et al. (2020), which shows that a similar two-price system is optimal in a specialised mechanism design setup. Dworczak et al. (2020) consider an economy in which, by assumption, the marginal utility of money differs across groups. We do not assume that marginal utility differs across groups, but we can endogenize this by considering a varied endowment distribution, one that leads to different consumption levels. Hence, we find a result similar to Dworczak et al. (2020) in spirit, if not form.

However, given that MS dominates QR in our construct, our results suggest that integrating certain markets (essentials) while segmenting it from others (non-essentials) can serve to improve welfare. Hence, we consider that the discussion in the market design literature might be enriched if it considers the welfare effects of integrating markets in general.[29] We thus believe that endogenizing the marginal utility of money and integrating markets might serve as a promising avenue for future research when considering market design.

Having discussed some applications of MS policy, we move on to discuss how it might be implemented. We discuss its implementation alone and in the context of a compli-

[28]For more detail on how the NHS works, please see their stated constitution https://www.gov.uk/government/publications/the-nhs-constitution-for-england/the-nhs-constitution-for-england.

[29]See Roth (2015) for a review of the market design literature.

mentary policy instrument of commodity taxation with direct transfers.

4.2 Implementation

In this subsection, we discuss how a social planner could choose to implement MS. It is important to note that the planner need not know individual endowments or the preferences of different people within the economy, but only their overall distribution. In this regard, the policy of MS and commodity taxation require equivalent information to be implemented.

However, to enforce segmentation, the planner must stop the rich from providing side payments to 'the dons' (*i.e., those owners of essential goods*). To enforce this, it is sufficient that the planner can link and monitor an individual's expenditure on essential goods. This can be done by linking social security or unique identification numbers to expenditure on essentials. Many social security and welfare schemes across the globe require such monitoring of individual levels of consumption. For instance, the 'consumption' of COVID-19 vaccines was linked to individuals and actively monitored. Similarly, the Public Distribution System (PDS) in India provides food grains on the principle of quantity rationing (see Gadenne (2020)), linking and monitoring household level purchases of such 'essential items'. Thus, monitoring might well be costly but nonetheless feasible given the available technologies at the disposal of the social planner.

Admittedly, implementing MS would require some cost of monitoring and the possibility of black market transactions. However, similar problems also exist in implementing such policy instruments as taxation and QR. Tax evasion is a well studied aspect of economics and a number of research papers note its high prevalence across many socioeconomic settings.[30] Moreover, tax compliance and revenue collection are costly.[31] Similarly, QR has its problems in terms of implementation, including high costs of administration and associated corruption.[32]

Importantly, examples related to our segmentary taxes exist in economies around the world, in which the tax rates depend both on the *source* of the income and the *type of goods* consumed using that income. For instance, in India, certain types of capital gain income (source) are taxed if they are used to consume such luxuries as a trip to Europe, yet are not taxed if employed to buy some other capital asset, for instance a house (type of goods).[33] Hence, tax incentives exist to invest in such assets as housing and government bonds. Even advanced countries, including the US, have similar provisions in their tax

[30]see Slemrod (2007)

[31]Pope (2002)

[32]The Public Distribution System (PDS) in India works on the principle of QR (see, Gadenne (2020)). Many have noted corruption in its implementation at various levels (see https://www.ndtv.com/india-news/corrupt-public-distribution-system-says-supreme-court-panel-412887.

[33]For details please refer to sections 54 and 54F of the Indian tax code (https://incometaxindia.gov.in/tutorials/15-%20ltcg.pdf).

codes.[34] However, taxes of this nature have not yet been theoretically explored as a policy instrument to deal with issues relating to inequality. In our setting, income accrued from the sale of luxury goods (source) is not taxed if it is used to buy other luxury items, yet it would be taxed if spent on essentials (type of goods). Hence, we feel optimistic that such taxes can be successfully implemented, especially given improving digital technology, enabling the state to actively monitor higher volumes of transactions.

Another possible issue with implementing MS could be a lack of a universal good in the real world. In the main model discussed in section 3, MS separates the market for essential and universal goods from that of non-essential goods. Since everyone is endowed with universal good in our construct, this creates an equitable distribution of all essential goods. However, it might be the case that there is no universal good in the economy. After all, manual labor might also exhibit heterogeneous productivity between individuals.

In such scenarios, MS can simply be implemented via *expenditure ceilings* on those goods classified as essential. By an expenditure ceiling, we mean that agents will not be taxed until their level of expenditure on essential goods falls below a certain threshold. However, if agents choose to spend more than the stipulated ceiling on essential goods, then they would have to pay a segmentary tax. The planner can just determine the Rawlsian income in the economy, *i.e.* that income earned by the least well-endowed individual within the economy and set that as an expenditure ceiling.[35] In effect, this policy will produce the same equilibrium outcomes as the model for a universal good as the product of the price of the universal good and its endowment determines the expenditure of essential goods by the agents. This will be equal to the Rawlsian income as described above.

Expenditure ceilings can also be determined as a function of the minimum wage within an economy. It is worth noting that using such expenditure ceilings allows planners to choose a threshold at a non-Rawlsian level of income. The optimum level of the ceiling will depend on the distribution of endowments within the economy and thus provide a potential topic of future research.

Finally, we note in this section that MS and commodity taxation with direct transfers are two policies that are fundamentally complementary in nature. Thus, the planner does not necessarily have to choose one over the other, but can rather implement them synergistically. We now present a theorem which proves that, unless one policy (MS or DT) in of itself achieves the first-best outcome, then the policies of MS and commodity taxation should be used together to improve social welfare within our construct.

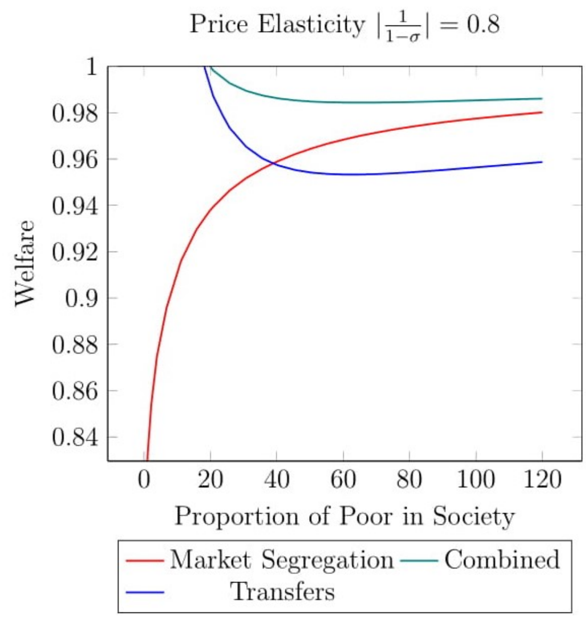
[34] see https://www.irs.gov/taxtopics/tc701.

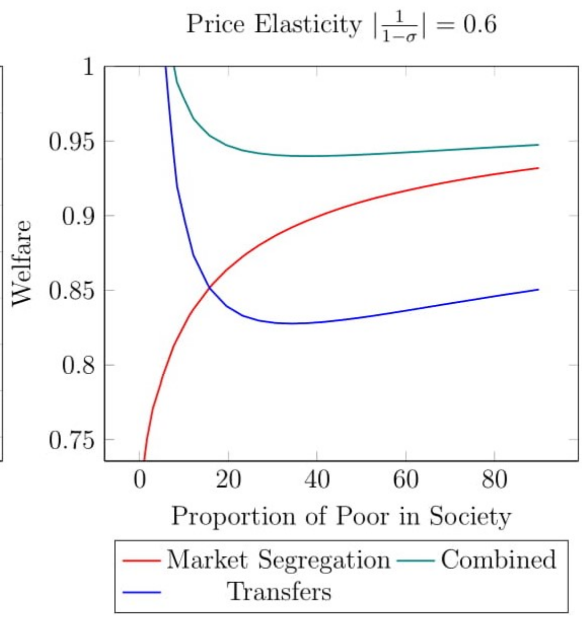
[35] The term 'Rawlsian income' follows the tradition of the social choice literature in which that social welfare function which maximizes the welfare of the least well-off individual is said to be Rawlsian (see https://plato.stanford.edu/entries/social-choice/#DocParDisDil.

Theorem 8. *Assume that the poor are consumption-constrained, even after implementing either segmentary taxes or commodity taxation with direct transfers. Using the other policy, in combination with the first, improves social welfare.*

Proof Idea. This is a natural outcome as DT and MS work to improve welfare via completely different approaches. Transfers increase the purchasing power of the poor, thus helping to create a more equitable consumption of essential goods. On the other hand, segmentation drives down the prices of these essential goods. Hence, the two operate synergistically. For a formal proof, see Appendix B.

An illustration of direct transfers and QR working well in concert has recently been observed in India. Despite a mediocre macroeconomic performance, India's poverty rate has declined rapidly. Some have attributed this to the welfare programs carried out by the Indian state.[36] The Indian state has combined direct transfers with QR to improve the welfare of the poor. We have previously discussed that MS is equivalent to QR in the case of there being only one essential good. Together, these facts suggest that some real-world evidence exists to support our theorem. We demonstrate this insight below with the help of figures plotting welfare on the y axis against the number of poor on the x-axis. The two figures illustrate that social welfare improves when transfers and MS are implemented in tandem.





[36] See discussion in Bhalla et al. (2022).

5 Conclusion

It is common for policymakers to intervene in a *laissez-faire* market equilibrium to improve social welfare, particularly to alter the final allocation of goods for redistributive purposes. This paper introduces a new policy instrument, termed MS, and compares it to existing instruments. Our model, though stylized, provides a systematic framework that allows us to think about how different policies can be more welfare-enhancing under different conditions.

For MS to work effectively, at least two types of goods must exist within the economy. In our model, there is one type of good which is distribution-relevant to the planner, namely essentials.[37] Second, there is another type of good (non-essentials) which are not relevant in terms of distribution. However, non-essentials being traded (or produced) is important for social welfare. Importantly, MS becomes relevant when there are at least two essential goods and two non-essential goods (moving from a 3-good to a 5-good economy in our example). The heterogeneity of agents' preferences, both for essential and non-essential goods, makes it relevant beyond the existing policy instruments of QR and taxation.

MS may not be the best policy even when the above conditions hold. Perhaps the main contribution of our work is to demonstrate that, under certain conditions, MS performs better than two commonly used policies, specifically taxation with subsidies and taxation with direct transfers. We also show that MS weakly dominates QR in our model.

Crucially, different policies' relative effectiveness depends on the price inelasticity of demand and supply of essential goods. The lack of response of demand and supply to price is what renders direct transfers and subsidies ineffective when compared to MS. Thus, policymakers need to carefully study the elasticity condition of the essential goods in question before making policy decisions.

Considering the price elasticity of demand to be inelastic might be a benign condition. However, the price elasticity of supply being inelastic is not, particularly in the long run. The supply of many essential goods may significantly respond to prices in the long run (*e.g.,* food grains). However, there might be many scenarios where supply is constrained due to technological constraints, even in the long run. For example, affordable housing in a city is constrained by the availability of land. Similarly, increasing the supply of cardiac surgeons might be tricky, even in the long run, due to the lengthy nature of training needed.[38] Hence, policymakers should be careful in applying our results to the real world. However, the complementarity of MS and DT as policy instruments is a particularly useful property for policymakers. Such complementarity implies that MS

[37]Formally, under separability, this means that the utility function for the essential good is concave and satisfies the Inada conditions.

[38]It takes some 16 years of training after high school to train a cardiothoracic surgeon in the USA and a similar time in Germany according to Tchantchaleishvili et al. (2010).

should be considered (alongside DT), even if the conditions we lay out are not entirely verifiable.

In our framework, we also demonstrate that transfers are better than subsidies under the same elasticity conditions discussed above. Under transfers, both rich and essential goods providers reduce their demand for essential goods, thereby leading to a more equitable distribution. However, under subsidies, only essential goods providers are affected. The inelasticity of demand implies that a large change in price for a few will not be as effective as a slight change in price for many. We also find that partial segmentation is better than complete segmentation because the segmentary tax revenue increases in the tax rate but jumps to zero in a discontinuous fashion at the complete segmentary level. We also consider different applications of our results and discuss how our model and results might have many real-world applications.

We believe that many, perhaps more sophisticated, instruments can be constructed using our method of taxing transactions between types of goods. Our paper can provide a bridge for promising future research to deal with questions related to inequality, taxation and even questions related to (dis)saving and investment.

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A Example

A.1 3 goods

Table 5: Endowments

Agents	x_1	x_2	x_3	x_4	x_5
A	1	1	e	0	0
B	0	0	e	0	0
C_1	0	0	e	W	0
C_2	0	0	e	0	W

Since everyone's utility is identical, maximization leads to the following demand:

$$\begin{aligned} x_1 &= \frac{k}{p_1} && if\ m \geq k \\ x_1 &= \frac{m}{p_1} && if\ m < k \\ x_2 = x_3 &= 0 && if\ m < k \\ x_2 + x_3 &= m - k && if\ m \geq k \end{aligned}$$

where $m = p_1.e_1 + p_2.e_2 + p_3.e_3$

For markets to clear, we require $p_2 = p_3$; we normalize this price to 1. Hence, $m_B = e$, $m_C = W + e$, and $m_A = p_1 + e$. If $1 < k < W + e$, then $x_1^B = \frac{e}{p_1}$ and $x_1^C = \frac{k}{p_1}$. Moreover, if in equilibrium, $m_A = p_1 + e > k$, $x_1^C = \frac{k}{p_1}$. Market clearing requires $x_1^A + nx_2^B + x_1^C = 1$. Hence, $p_1^* = ne + 2k$. (Note this implies $m_A > k$)

Market Segmentation (MS)

Under MS, the rich cannot use their endowment of luxury goods to purchase essentials. We now calculate social welfare under this policy intervention by finding the equilibrium. The utility maximisation exercise remains identical though under new endowments. Notably, the rich cannot use their wealth to buy health services as their endowment is identical to the poor.

As before, let $1 < k < W + e$. Type B and C will demand $x_1^B = \frac{e}{p_1}$, type A will still demand $x_1^C = \frac{k}{p_1}$[39]. Market clearing requires $x_1^A + nx_2^B + x_1^C = 1$. Hence, $p_1^* = k + ne + 1$ and the following are equilibrium allocations (note that this implies $m_A > k$.).

Table 6: Consumption under Regimes

Type	x_1^{LF}	x_1^{MS}
A	$\frac{e}{2k+e\pi_B}$	$\frac{e}{k+e(\pi_B+1)}$
B	$\frac{k}{2k+e\pi_B}$	$\frac{k}{k+e(\pi_B+1)}$
C	$\frac{k}{2k+e\pi_B}$	$\frac{e}{k+e(\pi_B+1)}$

The social welfare under this regime is

$$W^{MS} = (n+1)klog[\frac{1}{k+n+1}] + klog[\frac{k}{k+n+1}] + W + 3. \tag{3}$$

Welfare Comparison

We can now compare welfare under both regimes. For $W^{MS} > W^{LF}$, it must hold that:

$$(n+1)k\log\left[\frac{e}{k+n+1}\right] + k\log\left[\frac{k}{k+n+1}\right] > nk\log\left[\frac{e}{n+2k}\right] + 2k\log\left[\frac{k}{n+2k}\right]$$
$$\Leftrightarrow log\left[\frac{ke^{n+1}}{(n+1+k)^{n+2}}\right] > log\left[\frac{k^2e^n}{(n+2k)^{n+2}}\right].$$

Since the log is an increasing function, this implies that:

$$\frac{ke^{n+1}}{(n+1+k)^{n+2}} > \frac{k^2e^n}{(n+2k)^{n+2}}$$
$$\implies k(n+2k)^{n+2} > e(n+1+k)^{n+2}.$$

Assuming $e < k < W + 1$, the above inequality holds for a large enough n. Hence, MS improves social welfare for these values of k.

The rationale underlying such a result is simple. MS removes the wealth endowment from the economy and thus lowers the price of good 1 (health services) in equilibrium.

[39] assuming in equilibrium, $m_A = p_1 + e > k$

Thus, the poor can afford more of that good and their welfare improves. The welfare of the rich falls due to their inability to deploy their greater wealth in the pursuit of health services, yet that fall is offset by the gains of others for a sufficiently high value of n.[40]

A.2 5 Goods and Preference Heterogeneity

Welfare Comparison: Non-Distortionary Taxation

Welfare under non-distortionary taxes is given by: $klog\frac{k}{p_{DT}} + 2klog\frac{k}{p_{DT}} + nklog\frac{e+\frac{W}{2(n+3)}}{p_{DT}}$. Thus, for segmentation to be preferable, we need:

$$klog\frac{k}{p_{MS}} + (n+2)klog\frac{e}{p_{MS}} > klog\frac{k}{p_{DT}} + 2klog\frac{k}{p_{DT}} + nklog\frac{e+\frac{W}{2(n+3)}}{p_{DT}}$$
$$\leftrightarrow \log\left(\frac{k}{(p_{MS})^{n+3}}\right) > \log\left(\frac{k^3(1+\frac{W}{2(n+3)})^n}{(p_{DT})^{n+3}}\right)$$
$$\leftrightarrow \left\{\frac{p_{DT}}{p_{MS}}\right\}^{n+3} > k^2(e+\frac{W}{2(n+3)})^n$$

We can compute:

$$\frac{p_{DT}}{p_{MS}} = \frac{3ke+ne+W\frac{n}{2(n+3)}}{ne+k+2}$$
$$= \left\{1+\frac{2(k-e)+W\frac{n}{2(n+3)}}{ne+ke+2}\right\}$$

As n goes to infinity, the LHS

$$\left\{\frac{p_{DT}}{p_{MS}}\right\}^{n+3} \approx \left\{1+\frac{2(k-e)+W\frac{n}{2(n+3)}}{ne+ke+2}\right\}^{(n+3)e\times\frac{1}{e}} \rightarrow \left(e^{2(k-e)+\frac{W}{2}}\right)^{\frac{1}{e}}.$$

On the other hand, the RHS goes to 0 for small e. Thus, there exists some n, after which the LHS is greater than the RHS.

Welfare Comparison: Distortionary Taxation

Only essential goods and **untraded** luxury goods (which produce deadweight loss) are welfare relevant. We show that the welfare gain from using a distortionary tax regime over a non-distortionary tax regime is bounded. Furthermore, welfare loss increases with W. Therefore, for large values of n, MS outperforms distortionary taxation.

[40]The value of k effectively captures the importance of the essential good to an individual's welfare.

The distortionary price p is:

$$p = 2(1-\tau)k + k + ne + \frac{Rn}{n+3}.$$

The total welfare under distortionary taxation is:

$$W_{DT} = klog\frac{k}{p} + 2klog\frac{2(1-\tau)k}{p} + nklog\frac{1+\frac{R}{n+3}}{p}.$$

Welfare gain for the poor (over MS):

$$nlog\left[\frac{1+\frac{\left(4k\tau-\frac{2\tau}{1-\tau}\right)}{n+3}}{p}\right] - nlog\frac{1}{p_{MS}}.$$

Plugging in $p_{MS} = (n+2)e + k$ and simplifying, we get

$$\begin{aligned}
\Delta W &= nlog\left[\frac{(n+2)e+k}{ne+2(1-\tau)k+Rn/(n+3)} \times \left(1+\frac{4k\tau-\frac{2\tau}{1-\tau}}{n+3}\right)\right] \\
&= nlog\left(1+\frac{(2e+kt-Rn/(n+3)}{ne+2(1-\tau)k+Rn/(n+3)}\right) + nlog\left(1+\frac{4k\tau-\frac{2\tau}{1-\tau}}{n+3}\right).
\end{aligned}$$

The above expression is of the sort,

$$nlog(1+\frac{f_1}{n}) + nlog(1+\frac{f_2}{n}).$$

This bounded above, as n goes to infinity.

Thus, the welfare gain from using distortionary commodity taxation is bounded above. However, the dead weight loss due to this commodity taxation can be arbitrarily high and is independent of n. Deadweight loss in our example depends on W; thus, segmentation becomes better than taxation for a high enough W and n.

B Model: Proofs

Lemma 0.2: Under aggregate symmetry, the social planner treats essential goods symmetrically

Convexity of excess supply. First we show that the excess supply function is convex in tax rate. Using aggregate symmetry we show that the economy's set of achievable allocations is convex. Excess supply (as a function of tax rate τ or revenue (R)) is identical across goods and is given:

$$S(.) = \sum_{dons}(e_i - x_i)(p,\tau)$$
$$\frac{dS}{dR} = -\sum \frac{dx}{dp}\frac{dp}{dR}$$
$$\implies \frac{d^2S}{dR^2} = -\sum\left[\frac{d^2x}{dp^2}\left(\frac{dp}{dR}\right)^2 + \frac{dx}{dp}\frac{d^2p}{dR^2}\right].$$

To establish convexity, we must show $\frac{d^2S}{dR^2} \le 0$. We know $\left(\frac{dp}{dR}\right)^2 > 0$ (because it is a square term), and $\frac{dx}{dp} < 0$ because x is a normal good. We want to show that $\frac{d^2x}{dp^2} > 0$ and $\frac{d^2p}{dR^2} < 0$. Both of these conditions hold because of our inelasticity condition. See the proofs below:

1. $\frac{d^2p}{dR^2} < 0$. Proof:

$$p = p(m+1)x(p) + R + ne \implies \frac{dp}{dR} = \frac{dp}{dR}(m+1)x(p) + (m+1)p\frac{dx}{dp}\frac{dp}{dR} + 1$$
$$\implies \frac{dp}{dR} = \left[1 - (m+1)^{-1}(x(p) + p\frac{dx}{dp})\right]^{-1} > 0$$
$$\implies \frac{d^2p}{dR^2} = -\left[1 - (m+1)^{-1}(x(p) + p\frac{dx}{dp})\right]^{-2}\left[-(m+1)^{-1}\frac{dp}{dR}\frac{d}{dp}(p\cdot x)\right] < 0$$

2. $\frac{d^2x}{dp^2} > 0$. Proof:

$$u'(x) = \beta p \implies u''(x)\frac{dx}{dp} = \beta$$
$$\frac{dx}{dp} = \beta\frac{1}{u''} \implies \frac{d^2x}{dp^2} = -...u''' > 0$$

which gives us what we need.

From the preceding computations, we know that an economy's set of achievable allocations is convex. Now, we show if the set of achievable allocation is convex, then optimal allocations must be treated symmetrically by the planner. Let $(x_1, ...x_k)$ be an asymmetric allocation that is optimal (i.e., maximizes the social welfare function). By aggregate symmetry, for any i, j pair $x_i \neq x_j$, if $(x_1, ...x_i, .., x_j, .x_k)$ is feasible, so is $(x_1, ...x_j, .., x_i, .x_k)$. Now, because the set of individual allocations is convex, $(x_1, ...\frac{x_i+x_j}{2}, .., \frac{x_i+x_j}{2}, .x_k)$ must also be feasible. However, because the agents' utility functions are convex, these average allocations (which are feasible) increase social welfare. Hence, the original asymmetric allocation cannot be optimal. Thus, under aggregate symmetry, all optimal allocations must be symmetric.

Proof of Lemma 0.3

We have established that the planner must optimally treat all essential goods symmetrically.

Define:-

$$V^*_{sym} = \max_{\tau_1=\tau_2....\tau_n=\tau} V(\tau_1,,,,\tau_n)$$

and

$$V^* = \max_{\tau_1,...\tau_n} V(\tau_1,,,,\tau_n).$$

Notice that $V^*_{sym} \leq V^*$ because it is a maximization on a restricted set; however, by the above lemma, V^* must be such that $\tau_1 = \tau_2.... = \tau_n$. Thus, $V^*_{sym} \geq V^* \implies V^*_{sym} = V^*$. So, we can only analyze the maximization over the space $\tau_1 = \tau_2 = ...\tau_n$.

Further, $V^*_{sym} = \sum_i U_i(x_i(\tau_k))$. By aggregate symmetry, the prices of all goods must be equal, so we can consider the aggregate good (average) and the aggregate (average) consumer to compute social welfare.

B.1 Theorem 1

Theorem. *Under assumption 1 and significant gains from trade within luxury goods,* $\exists N$ *such that for all* $n \geq N$*, MS dominates DT.*

Let the consumption of essential goods by the don, the rich, and the poor be x_d, x_r, x_p. We show that essential good demand is just a function of its price.

Lemma 8.1. *Under our utilities (quasilinear) and the universal good taken as the numeraire, the demand for essential goods is independent of the prices of other goods.*

Proof. Follows quite simply from quasilinearity; see (Varian, 2014, p. 104). □

Claim. *[1.1] If assumption 1 holds,* $\exists N \in \mathbb{N}$*, such that, for* $n \geq N$*, social welfare in a regime where commodity taxation is high enough to restrict all trade in luxury goods dominates welfare for any non-distortionary tax regime.*

Proof. Welfare at the segmentary level of commodity taxation (τ_{DTS}) is given by:

$$(n+m)u(\frac{e}{p_{DTS}}) + u(x(p_{DTS})) - DWL = W_{DTS}.$$

Welfare under non-distortionary taxes is

$$nu(\frac{e}{p_{dt}}) + mu(x(p_{dt},\tau)) + u(x(p_{dt}) = W_{dt}.$$

Subtracting, we get:

$$n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + m(u(\frac{e}{p_{DTS}}) - u(x(p_{dt}, \tau))) + (u(x(p_{DTS})) - u(x(p_{dt}))) - DWL.$$

For this to be positive,

$$DWL < n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + m(u(\frac{e}{p_{DTS}}) - u(x(p_{dt}))) - (u(x(p_{DTS})) - u(x(p_{dt}))).$$

The RHS can be written as:

$$n\left[(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) + \frac{m}{n}((u(\frac{e}{p_{DTS}}) - u(x(p_{dt}, \tau)))) + \frac{1}{n}((u(x(p_{DTS})) - u(x(p_{dt}))))\right].$$

All terms are of the same order of magnitude, and n is increasing. Eventually, only the first term remains relevant. Hence, we get

$$\begin{aligned} n(u(\frac{e}{p_{DTS}}) - u(\frac{e}{p_{dt}})) &\approx nu'(\frac{e}{p_{dt}})\left[\frac{e}{p_{DTS}} - \frac{e}{p_{dt}}\right], \\ &= nu'(\frac{e}{p_{dt}})\frac{1}{n}\left[m(x_r(p_{dt}) - \frac{e}{p_{DTS}}) + x_d(p_{dt}) - x_d(p_{DTS})\right] \\ &= u'(\frac{e}{p_{dt}})\left[m(x_d(p_{dt}) - \frac{e}{p_{DTS}}) + x_d(p_{dt}) - x_d(p_{DTS})\right]. \end{aligned}$$

We know for a large n and a small e, $m(x_r(p_{dt}) - \frac{e}{p_{DTS}}) > \frac{e}{p_{dt}}$. We can use this to bound the above expression by $u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}} + x_d(p_{dt}) - x_d(p_{DTS})\right]$, which can be written as $u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}}\right] - u'(\frac{e}{p_{dt}})\left[x_d(p_{DTS}) - x_d(p_{dt})\right]$.

The first term is unbounded. Notice that $u'(\frac{e}{p_{dt}})\left[\frac{e}{p_{dt}}\right]$ can be written as $p^*x(p^*)$, where p^* is the price which supports $\frac{e}{p}$ or $x^{-1}(\frac{e}{p})$. Further, as $\frac{e}{p}$ goes to zero, its inverse goes to infinity, which means $p^*x(p^*)$ is unbounded by the elasticity condition.

We now show that the second term is bounded. Observe that $'(\frac{e}{p_{dt}})\left[x_d(p_{dt}) - x_d(p_{DTS})\right] \approx u'(\frac{e}{p_{dt}})\frac{dx}{dp}(p_{dt} - p_{DTS})$. Further, at the segmentation price, $u'(\frac{e}{p_{dt}}) = \lambda p_{dt}$, which gives us $\lambda p_{dt}\frac{dx}{dp}(p_{dt} - p_{DTS})$. The inelasticity condition gives us $\frac{d}{dp}(px) \geq 0$ which implies that $p\frac{dx}{dp} \leq x(p)$ .This can be substituted into the above expression to give:

$$x(p_{dt})(p_{dt} - p_{DTS}) = \frac{e}{p_{dt}}(p_{dt} - p_{DTS}),$$

which is bounded. Thus, for large values of n,$RHS > 0$, giving us what we need. □

Claim. *[1.2] Under assumption 1, there exists a distortionary level of taxation* τ^*_{DT} *which achieves higher welfare than* τ_s.

Proof. Let τ_{DTS} be the complete segmentation level of the commodity tax. A slight decrease in the tax rate from this level has three effects on welfare. First, decreasing the tax rate raises some tax revenue for the planner, which can be used as a direct transfer to improve welfare. This is given by the following: $\frac{dW}{dT} \times \frac{\delta T}{\delta \tau}$.

On the other hand, the decrease in the tax rate causes the price of essential goods to rise for the poor, thereby decreasing welfare. This is given by $\frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}$. Third, the dead weight loss in the economy also decreases. This is given by $\frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau}$.

Thus, what we want to show is that as we go from τ_{DTS} to $\tau_{DTS} - \epsilon$, where $\epsilon > 0$ but arbitrarily small,

$$\frac{dW}{dT} \times \frac{\delta T}{\delta \tau} + \frac{dW}{dDWL} \times \frac{\delta DWL}{\delta \tau} > \frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}. \tag{4}$$

Close to the complete segmentation tax rate, (*i.e.,* $\tau_{DTS} - \epsilon$), the change in demand is small. However, the income raised close to τ_{DTS} is both large and positive. This is because demand is a continuous function of price, and price is a continuous function of the tax rate. Thus, demand is continuous in prices. However, if the good is inelastic, the total expenditure on the commodity is an increasing function of price (and tax rate) and thus tax revenue, which is an increasing function of tax rate util τ_{DTS}, where it falls discontinuously to zero. Hence, as $\tau \to \tau - \epsilon$ then $\frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \to 0$ but $\frac{\delta T}{\delta \tau} \to R >> 0$. Now, given that $\frac{dW}{dT} > 0$, 4 holds. $\square$

Claim (1.3)**.** *Let the welfare gain generated by revenue of R, where the relative number of poor is n, be $G_n(R)$. Then:*

$$\exists \quad G^*(R) \quad \textit{such that} \quad \forall n \quad G_n(R) \leq G^*(R)$$

Proof. If the planner extracts revenue R by tax τ, the total welfare of the economy is:

$$W* = n(U(\frac{e + \frac{R}{n+3}}{p*}) + mU(x(p*, \tau)) + U(x(p*)) - DWL_{DT*}.$$

Comparing this to the complete segmentation tax rate, we get:

$$\begin{aligned} W* - W_{DTS} = n(U(\frac{e + \frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + m(U(x(p*, \tau)) - U(p_{DTS}, \tau)) + \\ U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}. \end{aligned}$$

It is clear that $m(U(x(p*, \tau)) - U(p_{DTS}, \tau)) + U(x(p*)) - U(p_{DTS}) - DWL_{DT*} + DWL_{DTS}$ is bounded, we call it M. Thus, we get:

$$\begin{aligned} & n(U(\frac{e + \frac{R}{n+3}}{p*}) - U(\frac{e}{p_{DTS}})) + M \\ =& nu'(\frac{e}{p_{DTS}})\frac{1}{n} [1 - mx(p*, \tau) - x(p*) - (1 - mx(p_{DTS}, \tau) - x(p_{DTS}, \tau))] + M. \end{aligned}$$

As a utility change can be approximated as the marginal utility multiplied by the change in consumption, we get:

$$\begin{aligned}\Delta U &= u'(\frac{e}{p_{DTS}})\left[-m(x(p*,\tau)+x(p_{DTS},\tau)+x(p_{DTS})-x(p*))\right]+M\\ &= u'(\frac{e}{p_{DTS}})\left[-m\frac{dx}{d(p*(1-\tau))}(p*(1-\tau)-p_{DTS})-\frac{dx}{dp}(p*-p_{DTS})\right]+M.\end{aligned}$$

At the segmentation level of commodity taxation, the marginal utility of the poor must be equal to the marginal utility of the rich, *i.e.* $u'(\frac{e}{p_{DTS}})=\lambda p_{DTS}$

$$=\lambda p_{DTS}\left[-m\frac{dx}{d(p*(1-\tau))}(p*(1-\tau)-p_{DTS})-\frac{dx}{dp}(p*-p_{DTS})\right]+M$$

This expression, which represents the gain, is bounded by the in-elasticity condition. □

B.2 Theorem 2

Theorem. *Under assumption 1, given any revenue R,* $\exists N\in\mathbb{N}$, *such that* $\forall\quad n\geq N$, *direct transfers are more efficient than subsidies on essential goods.*

Proof. We can express welfare under transfers (*i.e.* W_{DT}) as:

$$nu(\frac{1+\frac{R}{n+m+1}}{p_{DT}})-nu(\frac{1}{p_{LF}})+mu(x_r(p_{DT}))-mu(x_r(p_{LF}))+u(x_d(p_{DT}))-u(x_d(p_{LF})).$$

Prices are thus;

$$\begin{aligned}p_{LF} &= \frac{n}{1-(m+1)x_{LF}}\\ p_{DT} &= \frac{n+R\frac{n}{n+m+1}}{1-mx_r(p_{DT},\tau)-x_d(p_{DT},\tau)}\end{aligned}$$

In any such equilibrium, the following equation must hold:

$$\begin{aligned}1 &= x(p_s+\Delta p)+\frac{n}{p_s}+mx(p_s)\\ \implies p_s &= \frac{n}{1-mx(p_s)-x(p+\Delta p)},\end{aligned}$$

where p_s is the price in the subsidy regime and Δp is the price paid by the government to the don on each unit sold. Therefore, under a balanced budget:

$$\begin{aligned}R &= (1-x_{ds})\Delta p\\ \implies p_s &= \frac{n}{1-mx(p_s)-x(p+\frac{R}{1-x_{ds}})},\end{aligned}$$

where x_{ds} is the consumption of dons in the subsidy regime. Further,

$$W_S = nu(\frac{1}{p_s}) - nu(\frac{1}{p_{LF}}) + mu(x_r(p_s)) - mu(x_r(p_{LF})) + u(x_d(p_s)) - u(x_d(p_{LF})).$$

Comparing both regimes

$$\begin{aligned} W_{DT} - W_S = n\Big[u\Big(\frac{1}{n}(1 - mx_r - x_d)\Big) - u\Big(\frac{1}{n}(1 - mx(p_s) - x(p + \frac{R}{1 - x_{ds}}))\Big)\Big] \\ + m\Big[u(x_r(p_{DT})) - u(x_r(p_s))\Big] + \Big[u(x_d(p_{DT})) - u(x_d(p_s))\Big]. \end{aligned}$$

All terms are of the same order of magnitude, and n is increasing. Eventually, only the first term remains relevant. Hence, for a large n;

$$W_{DT} - W_S = n\Big[u'(.)\Big(\frac{1}{n}(1 - mx_r - x_d) - \frac{1}{n}(1 - mx_r(p_s) - x_d(p_s))\Big)\Big]. \tag{5}$$

As $u'(.) > 0$, to show that $W_{DT} - W_{MS} > 0$, we need to show that:

$$m(x(p_s) - x(p_{DT})) + (x_r(p_s + \frac{R}{1 - x_d(p_s)}) - x_r(p_{DT})) > 0$$

The LHS can be approximated by

$$\begin{aligned} & m\frac{dx}{dp}|_{p=p_{DT}}(p_s - p_{DT}) + \frac{dx}{dp}|_{p=p_{DT}}(p_s + \frac{R}{1 - x_d} - p_{DT}) \\ & = \frac{dx}{dp}|_{p=p_{DT}}\Big[(m + 1)(p_s - p_{DT}) - \frac{R}{1 - x_d}\Big] \end{aligned} \tag{6}$$

Notice that $p_s < p_{LF}$, which means that:

$$p_{DT} - p_s > p_{DT} - p_{LF} = \frac{n + R\frac{n}{n+m+1}}{1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau)} - \frac{n}{1 - (m + 1)x_{LF}}.$$

Given that $x_r(p_{DT}, \tau) < x_r(p_{LF})$ and $x_d(p_{DT}, \tau) < x_d(p_{LF})$, we know that:

$$p_{DT} - p_s > p_{DT} - p_{LF} > R\frac{n}{n + m + 1}\frac{1}{(1 - mx_r(p_{DT}, \tau) - x_d(p_{DT}, \tau))(1 - (m + 1)x_{LF})}.$$

For a large n, the above equation implies:

$$p_s - p_{DT} \geq R.$$

Putting this back into equation 6, we get:

$$\frac{dx}{dp}|_{p=p_{DT}}\left[(m+1)R-\frac{R}{1-x_d}\right]$$

We have found that direct transfers are better than subsidies for a large n as long as $m+1>\frac{1}{1-x_d}$. Also, notice that as $n\to\infty, x_d\to 0$. Thus, the condition requires $m>0$, i.e., there is a positive number of rich people in the economy. $\square$

Subsidies only decrease the consumption of the 'dons'. However, direct transfers decrease the consumption of both the rich and the dons, making direct transfers more efficient than subsidies if the above conditions hold.

B.3 Theorem 3

Theorem. *Consider an economy with two essential goods and idiosyncratic preference heterogeneity. In this setup, MS improves the welfare of the poor more than the quantity rationing of essential goods.*

Proof. Suppose there are two *essential goods*, x^1 and x^2 and preference heterogeneity is indexed by the parameter $\omega\in[0,1]$. Thus, the utility function can be described by:

$$U_\omega(X)=\omega u(x^1)+(1-\omega)u(x^2)+\sum b_i x_i$$

Preferences in the population are described by a density $f(\omega)\in\Delta[0,1]$. Under our assumption of aggregate symmetry, we know that $f(\omega)=f(1-\omega)$. Let the endowment of the universal good be e.

The symmetry of $f(\omega)$ implies that the equilibrium price p of x^1 and x^2 must be equal under MS. By market clearing:

$$x_d^i(p)+n\mathbf{E}_{\omega\sim f}[x_p^{i\omega}(p)]+m\mathbf{E}_{\omega\sim f}[x_r^{i\omega}(p)]=1 \quad \text{for} \quad i=1,2$$

Adding equations for $i=1,2$ gives us:

$$x_d^1(p)+n\mathbf{E}_{\omega\sim f}[x_p^{1\omega}(p)]+m\mathbf{E}_{\omega\sim f}[x_r^{1\omega}(p)]+x_d^2(p)+n\mathbf{E}_{\omega\sim f}[x_p^{2\omega}(p)]+m\mathbf{E}_{\omega\sim f}[x_r^{2\omega}(p)]=2$$

Under MS, both the poor and the rich can only use the universal good to buy essential goods. So, $x^{1\omega}(p)+x^{2\omega}(p)=\frac{e}{p}$. Multiplying the above equation by p,

$$p(x_d^1(p)+x_d^2(p))+(n+m)e=2p \implies p=\frac{(n+m)e}{2-x_d^1(p)-x_d^2(p)}.$$

At these prices, individuals buy essential goods depending on their respective ω.

Suppose the planner attempted to mimic the outcome of MS via a QR policy. A non-linear tax can implement any rationing policy[41] via an infinite tax on consumption above a certain level $\bar{q}$.

If the poor have the same utility under rationing, it must be that $\bar{q}_1 = \bar{q}_2 = \frac{e}{p_{ms}} = \frac{2-x_d^1(p)+x_d^2(p)}{(n+m)}$. However, if $\bar{q}$ is as above, then the aggregate consumption of some of the rich is not $m\mathbf{E}_{\omega\sim f}[x_\omega(p)]$. It lies between $m\mathbf{E}_{\omega\sim f}[x_\omega(p)]$ and $m\bar{q}$ for each essential good, where $\bar{q} >> \mathbf{E}_{\omega\sim f}[x_\omega(p)]$. Thus, we have a situation of excess demand in the market, and the market does not clear. Thus, the planner has to set a new $\bar{q}_n \lneq \bar{q}$ to clear the market. As discussed above, this makes the poor worse off. □

B.4 Theorem 4

Theorem. *Under assumption 1, partial MS dominates complete MS.*

Proof. Let τ^{cms} be the complete segmentation level of the tax. A slight decrease in tax rate firstly raises tax revenue, which can be used as a transfer. This gain is given by $\frac{dW}{dR} \times \frac{\delta R}{\delta \tau}$. The decrease also raises the price of essential goods, in turn decreasing welfare. This is given by $\frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}$.
Let $\epsilon > 0$ be small. We want to show as we go from τ^{cms} to $\tau^{cms} - \epsilon$:

$$\frac{dW}{dR} \times \frac{\delta R}{\delta \tau} > \frac{dW}{dx_{ess}} \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau}. \tag{7}$$

Notice that close to the complete segmentation tax rate (*i.e.*, $\tau^{cms} - \epsilon$), the change in demand is small. However, the income raised close to τ^{cms} is large and positive, using an argument analogous to Claim 1.2. Hence, $\tau \to \tau - \epsilon \implies \frac{\delta x_{ess}}{\delta p} \frac{\delta p}{\delta \tau} \to 0$. However, $\frac{\delta T}{\delta \tau} \to R >> 0$. As $\frac{dW}{dT} > 0$, 7 must hold. □

B.5 Theorem 5

Theorem. *At the optimal segmentary tax rate* (τ_s)*,*

$$\pi\lambda_r\left[\frac{\partial R_{MS}}{\partial \tau}(1+\tau)-\left(p_1(x_e-e_e)+p_2(x_u-e_u)-R_{MS}\right)\right]+\left[\pi_d\lambda_d+\pi_p\lambda_p\right]\frac{\partial R_{MS}}{\partial \tau} = \sum \mu_k \sum \frac{dx(p,\tau)}{d\tau}$$

or

$$\frac{\partial R}{\partial \tau_s}\left[\lambda_r(1+\tau)\pi_r + \lambda\pi_d + \pi_p\lambda_p\right] - \sum \mu_k \sum \frac{dx(p,\tau_s)}{d\tau_s} = \pi_r\lambda_r\left[\left(p_e(x_e-e)-(x_u-e_u)-R_{MS}\right)\right].$$

π_r, π_d, and π_p are the weights of the rich, the dons, and the poor. x_e, x_u is the essential and universal good consumption of the rich. R_{MS} is the per capita transfer, λ

[41] see, Gadenne (2020) for details

is the marginal utility of income for each group, and μ is the multiplier associated with the resource constraint.

Proof. The planner's problem is to maximize the following expression:

$$\pi_p V_p(p,\tau) + \pi_d V_d(p,\tau) + \pi_r V_r(p,\tau) \tag{8}$$

with respect to the segmentary tax rate τ, and subject to

$$\sum x_i(p,\tau) = \sum e_i \forall k, \tag{9}$$

where V_p, V_r, V_d are the indirect utility functions of the poor, the rich, and the dons. Each k represents a good, and π_i is the proportion of each type. The Planner's Lagrangian is:

$$L(.,\tau) = \pi_p V_p(p,\tau) + \pi_d V_d(p,\tau) + \pi_r V_r(p,\tau) + \sum \mu_k \Big[\sum e_i - \sum x_i(p,\tau)\Big].$$

Differentiating L with respect to τ, we get:

$$\pi_p \frac{dV_p}{d\tau} + \pi_d \frac{dV_d}{d\tau} + \pi_r \frac{dV_r}{d\tau} = \sum \mu_k \sum \frac{dx(p,\tau)}{d\tau}. \tag{10}$$

Now, we carry out utility maximization exercises to find expressions of the indirect utility for each type.

The market is segmented, i.e., essential and universal goods are in one sub-market, and luxury goods are in another. Also, suppose there is an asset (y) introduced by the planner. This asset enables people to transfer income between markets at a tax rate of τ. This tax generates revenue, distributed to all individuals at per capita transfer R_{MS}. The utility maximization problem (UMP) of the rich:

$$\begin{aligned} \underset{x}{\text{maximize}} \quad & U_r(x) \\ \text{subject to} \quad & p_1(x_1 - e_1) + p_2(x_2 - e_2) \le y + R_{MS} \\ & \sum p_k(x_k - e_k) \le -(1+\tau)y. \end{aligned}$$

Assuming that both equations bind,

$$(1+\tau)\left[p_1(x_1 - e_1) + p_2(x_2 - e_2) - R_{MS}\right] + \sum p_k(x_k - e_k) \le 0.$$

Using the envelope theorem and differentiating $V_r(p,\tau)$ with respect to τ:

$$\frac{\partial V}{\partial \tau} = \frac{\partial L}{\partial \tau} = -\lambda_r \Big[(p_1(x_1 - e_1) + p_2(x_2 - e_2) - R_{MS} - \frac{\partial R_{MS}}{\partial \tau}(1+\tau)\Big].$$

The don doesn't own any luxury, so his UMP can be formulated as:

$$\begin{aligned}
&\underset{x}{\text{maximize}} && U_d(x) \\
&\text{subject to} && p_1(x_1 - e_1) + p_2(x_2 - e_2) - R_{MS} + \sum p_k(x_k - e_k) \leq 0.
\end{aligned}$$

So:

$$\frac{dV_P}{d\tau} = -\lambda_d(-\frac{\partial R_{MS}}{\partial \tau}).$$

The problem for the poor:

$$\begin{aligned}
&\underset{x}{\text{maximize}} && U_p(x) \\
&\text{subject to} && p_1(x_1 - e_1) + p_2(x_2 - e_2) - R_{MS} \leq 0.
\end{aligned}$$

Analogous to the first two types,

$$\frac{dV_P}{d\tau} = -\lambda_p(-\frac{\partial R_{MS}}{\partial \tau}).$$

Putting the above values into equation 10 gives us the following:

$$\pi\lambda_r\left[\frac{\partial R_{MS}}{\partial \tau}(1+\tau) - \left(p_1(x_1-e_1)+p_2(x_2-e_2)-R_{MS}\right)\right] + \pi_d\lambda_d\frac{\partial R_{MS}}{\partial \tau} + \pi_p\lambda_p\frac{\partial R_{MS}}{\partial \tau} = \sum \mu_k \sum \frac{dx(p,\tau)}{d\tau},$$

which can be rearranged to fetch what we need:-

$$\frac{\partial R}{\partial \tau_s}\left[\lambda(1+\tau)\pi_r + \lambda\pi_d + \pi_p\lambda_p\right] - \sum \mu_k \sum \frac{dx(p,\tau_s)}{d\tau_s} = \pi_r\lambda\left[\left(p_e(x_e-e)-(x_u-e_u)-R_{MS}\right)\right].$$

□

B.6 Theorem 6

Theorem. *As the weight of the universal good in the utility function falls (i.e. $b \to 0$), direct transfers are more effective in improving social welfare than MS.*

Claim (6.1)**.** *Let b be the weight associated with the universal good (x_2) in individuals' utility functions, and let e be the (common) endowment of the universal good. As $b \to 0$, the equilibrium allocation produces the same welfare as $e \to 0$.*

The utility functions in the economy can be given by:

$$U = u(x_1) + bx_2 + \ldots \quad .$$

The first-order conditions of the rich and the dons give us

$$\frac{u'(x_1)}{b} = \frac{p_1}{p_2} \implies u'(x_1) = \frac{p_1}{p_2} b.$$

The utility maximization problem of the poor gives us:

$$x_1 = \frac{p_2 \times e}{p_1}$$

Define $p^* = \frac{p_1}{p_2} b$. The following three equations uniquely determine the consumption of the rich and poor:

$$u'(x) = p^*$$
$$x_p = \frac{b.e}{p^*}$$
$$\sum x_i = 1.$$

$u'(x)$ is independent of b and $x > 0$, $p^* > 0$. Thus, in equilibrium;

$$b \to 0 \implies x_p \to 0 \iff e \to 0 \implies x_p \to 0.$$

Claim (6.2)**.** *As the endowment of the universal good (e) goes to zero, the ratio of the marginal utilities of the poor to the rich goes to infinity.*

Proof. The marginal utility of the essential good for the rich is λp where λ is the marginal utility of income. The ratio of the marginal utility of the poor to that of the rich is:

$$\frac{u'(\frac{e}{p})}{\lambda p}.$$

As $e \to 0$, λp is bounded. However, $e \to 0 \implies u'(0) \to \infty$, which implies $\frac{u'(\frac{e}{p})}{\lambda p} \to \infty$. □

Claim (6.3)**.** *Considering the welfare of the poor, as the endowment of the universal good (e) goes to zero, direct transfers dominate complete MS.*

Proof.

$$W_{DT} = n \frac{e + \frac{T}{n+m+1}}{p_{DT}}, \tag{11}$$

where

$$p_{DT} = \frac{en + T\frac{n}{n+m+1}}{1 - (m+1)x_{DT}},$$

and x_{DT} is the consumption of dons and the rich under transfers.

Under MS:

$$W_{ms} = (n+m)\frac{e}{p_{ms}}, \tag{12}$$

$$p_{ms} = \frac{e(n+m)}{1 - x(p_{ms})}.$$

Here, x_{ms} is the consumption of don under MS. Subtracting the 12 from 11, we get:

$$W_{DT} - W_{ms} = 1 - (m+1)x(p_{DT}) - (1 - x(p_{DT})) = x(p_{ms}) - (m+1)x(p_{DT}). \quad (13)$$

As $e \to 0$, we know that consumption of the dons under MS goes to 1 because both the rich and the poor are unable to demand anything when their endowment goes to zero.

On the other hand, under direct transfers, the poor do receive positive transfers and thus demand a strictly positive amount of essential goods even when their endowment goes to zero. Thus, the consumption of the don and the rich people combined is bounded strictly below 1. Hence, 13 is strictly positive. □

B.7 Theorem 7

Theorem. *As the value of the universal endowment good (b) goes to zero, direct transfers are more effective than subsidies on the essential good.*

Proof. The first two parts of the proof are the same as claims 6.1 and 6.2. The next claim concludes the proof.

Claim. *As the endowment of the universal good in the economy goes to zero, direct transfers are more effective than subsidies.*

Welfare under direct transfers can be given by

$$W_{DT} = n \frac{\epsilon + \frac{R}{n+m+1}}{p_{DT}}, \quad (14)$$

where

$$p_{DT} = \frac{\epsilon n + R\frac{R}{n+m+1}}{1 - (m+1)x_{DT}},$$

and x_{DT} is the consumption of dons and the rich under transfers.

Under subsidies,

$$W_S = n\frac{\epsilon}{p_s}, \quad (15)$$

where

$$p_s = \frac{\epsilon n}{1 - m.x(p_s) - x_{ds}(p + \frac{R}{1-x_{ds}})},$$

and x_{p_s}, x_{d_s} is the consumption of the rich and dons under subsidies.

Subtracting the two welfare equations above, we get:

$$W_{DT} - W_S = 1 - (m+1)x_{DT} - \left(1 - x(p_s) - mx(p_s + \frac{R}{1 - x_{ds}})\right) \quad (16)$$

$$= mx(p_s) + x(p_s + \frac{R}{1 - x_{ds}}) - (m+1)x(p_{DT}).$$

As $e \to 0$, consumption of the dons and rich, *i.e.* $mx(p_s) + x(p_s + \frac{R}{1-x_{ds}})$ goes to 1. They consume everything as the poor cannot demand anything, even with a subsidy.

On the other hand, under direct transfers, poor people do receive positive transfers and thus demand a strictly positive amount of essential goods. Thus, the consumption of the don and the rich people combined is bounded strictly below 1. Hence, 16 is strictly positive. An argument analogous to the previous theorem and the above claim proves this theorem. □

B.8 Theorem 8

Theorem. *Suppose the poor are consumption constrained after implementing either segmentary taxes or commodity taxation with direct transfers. Using the other policy with the first improves social welfare.*

Proof. First, we show that under a regime with only commodity taxation and direct transfers, some positive value of segmentary taxes benefits the poor. By market clearing:

$$1 = x_d(p) + nx_p(p) + mx_r(p). \tag{17}$$

$x_d(p), x_p(p)$, and $x_r(p)$ represent the consumption of the essential good by the don, the poor, and the rich.

Suppose the planner sets a segmentary tax high enough to increase the price of the essential good for the rich[42]. At this level of segmentary tax, the consumption of the rich decreases. By market clearing:

$$1 = x_d(p + \delta p) + nx_p(p + \delta p) + m(x_r(p) - \delta S) \tag{18}$$

where δS is the change in rich consumption due to segmentation.

Subtracting 17 from 18, we obtain:

$$\left(x_d(p + \delta p) - x_d(p)\right) + n\left(x_p(p + \delta p) - x_p(p)\right) = m\delta S. \tag{19}$$

As the RHS of the equation is positive, so must be the LHS, meaning δp must be negative. Thus, the welfare of the poor, which is $\frac{e + \frac{T}{n+m+1}}{p}$, increases under this policy.

Now, we show that the converse. If any regime with segmentary taxes does not achieve the first best outcome, commodity taxation benefits the poor if it generates any revenue.

[42] We know such a tax exists because a complete segmentation level of tax will always exist that will definitely increase the price for the rich

The consumption of the poor in the segmentation economy can be given by $[1 - x_d(p_{ms}) - mx_r(p_{ms})]$, where $x_d(p_{ms})$ and $x_r(p_{ms})$ is the consumption of the don and the rich. We want to show that:

$$[1 - x_d(p_{ms} + \delta p) - mx_r(p_{ms} + \delta p)] > [1 - x_d(p_{ms}) - mx_r(p_{ms})], \tag{20}$$

where δp is the change in price after commodity taxation and direct transfers. So, we need to show that for any price p and endowment e, the introduction of direct transfers increases the price, *i.e.* $\delta p > 0$, which concludes the above proof.

Let us suppose that in the segmentary equilibrium, the poor have income i[43], and the commodity price is p. By market clearing:

$$p = px_d + ni + mpx_r.$$

Now, suppose we provide direct transfers of R to the population. We get:

$$p_{DT} = p_{DT}x_d(p_{DT}) + p_{DT}mx_r(p_{DT}) + ni + R\frac{n}{n+m+1}.$$

Implicitly differentiating this equation with respect to R:

$$\frac{dp}{dR} = \frac{n}{n+m+1} + [mx_r + x_d]\frac{dp}{dR} + p[\frac{mdx_r}{dp} + \frac{dx_d}{dp}]\frac{dp}{dR}. \tag{21}$$

This means that

$$\frac{dp}{dR} = \frac{1}{(1 - [mx_r + x_d] - p[\frac{mdx_r}{dp} + \frac{dx_d}{dp}])}\frac{n}{n+m+1}. \tag{22}$$

The above expression is positive because $mx_r + x_d < 1$ and $[\frac{dX_r}{dp} + \frac{dx_p}{dP}] < 0$. Thus, increasing R always increases price p. So, the new welfare for the poor, which is $1 - x_r(p + \delta p) - x_d(p + \delta p)$, is higher than the old. □

C Analyzing MS in a Labor Supply Model

We augment the basic discrete job model of Saez (2002) with a distinction between two *types* of consumption goods. The first type (luxury goods) are produced using labor and CRS technology. The second type (essential goods) are endowed to a few people, called 'dons.' We assume aggregate symmetry; thus, we can reduce the problem by considering only one essential good.

[43]This includes the transfers received from tax revenue generated from segmentary tax

C.1 Goods and Production

Our economy has two goods: essential (x_e) and non-essential (x_l). 'Dons' are endowed with 1 unit of x_e and do not provide labor for production. There are two jobs: high and low productivity, and they both produce luxury goods.

$$y(L) = \begin{cases} c_l L & \text{at the low productivity job} \\ c_h L & \text{at the high productivity job} \end{cases}$$

We assume that wages equal marginal products. Thus, individuals working in high and low productivity jobs earn $w_h = c_h$ and $w_l = c_l$.

C.2 Individuals

Individuals, denoted by ω, have identical utilities but idiosyncratic ability $\theta_\omega \in [0, h]$. We interpret θ_ω as the effort ω exerts to do the high-productivity job. Each ω is endowed with 1 unit of labor. Let $f(.)$ and $F(.)$ be the PDF and CDF for θ_ω.

Individuals take prices as given and use wages to buy consumption. We define the utility of individual ω as:

$$U^\omega(x_l, x_e) = \begin{cases} u(x_e) + x_l & \text{at the low productivity job} \\ u(x_e) + x_l - \theta_\omega & \text{at the high productivity job} \end{cases}$$

x_l and x_e represent luxuries and essentials, and u satisfies the INADA conditions. For the sake of exposition, we assume that u exhibits a CES-like utility structure:

$$u(x) = \frac{x^\sigma}{\sigma} \quad \text{where} \quad \sigma < 0$$

The reader should note that $\sigma < 0$ corresponds to inelastic demand, $\sigma > 0$ corresponds to the elastic case, and the limit as $\sigma \to 0$ corresponds to the isoelastic log utility case. In this setup, essential good demand for the rich is given by:

$$x_e(p) = p^{\frac{1}{\sigma-1}}.$$

Expenditure on the good is given by:

$$px_e(p) = p^{\frac{\sigma}{\sigma-1}},$$

an increasing and unbounded function of price.[44]

[44] As in the main model, this is what we need: inelasticity implies that the expenditure function is increasing in price.

C.3 Laissez Fair (LF) Equilibrium

Let $V(c,p)$ denote the utility of agent ω with $\theta_\omega = 0$ at wage c and essential good price p. Their utility of taking the high-productivity job is $V_\omega(h,p) = V(c_h,p) - \theta_\omega$ and the low-productivity job is $V_\omega(l,p) = V(c_l,p)$. Individual ω takes a high-productivity job if and only if:

$$V(c_l,p) < V(c_h,p) - \theta_\omega \iff \theta_\omega < V(c_h,p) - V(c_l,p).$$

Define m as a measure of people who take up the high productivity job LF:

$$m = \int_0^{V(c_h,p)-V(c_l,p)} f(\theta)\, d\theta.$$

Similarly, n is defined as a measure of people taking up low-productivity jobs:

$$n = \int_{V(c_h,p)-V(c_l,p)}^{h} f(\theta)\, d\theta.$$

We assume that $c_h > a$ and $c_l < a$, where a is the satiation expenditure on the essential good. Thus, the consumption of the essential good of the high-productivity job takers (rich) and the dons is $x_e(p)$, and the consumption of the low-productivity job takers (poor) is $\frac{c_l}{p}$. By market clearing:

$$n\frac{c_l}{p_{LF}} + mx_e(p_{LF}) + x_e(p_{LF}) = 1.$$

The above equation gives us the following equilibrium price:

$$p_{LF} = \frac{1}{\big(1 - (m+1)x_e(p_{LF})\big)} nc_l.$$

C.4 Planner's Intervention

Now, suppose that the planner wants to intervene in the economy to maximize the utilitarian objective $W = \int_\omega U^\omega f(\omega) d\omega$. We now compare two interventions: non-linear labor taxation and, segmenting the market for essential goods. We first discuss taxation.

C.4.1 Non-Linear Taxation

In our setup, the planner can tax high-productivity job holders and give transfers to low-productivity job holders. Let t_h be the tax at the high-productivity job and t_l the transfer at the low-productivity job. If individual ω takes a high-productivity job, they get the utility:

$$V_\omega(c_h - t_h, p) = V(c_h - t_h, p) - \theta_\omega.$$

In the low-productivity job, they get:

$$V_\omega(c_l + t_l, p) = V(c_l + t_l, p).$$

An individual takes a high-productivity job if and only if:

$$V(c_l + t_l, p) < V(c_h - t_h, p) - \theta_\omega \iff \theta_\omega < V(c_h - t_h, p) - V(c_l + t_l, p).$$

Define $m(t)$ as the measure of high-productivity workers at tax rate t:

$$m(t_h) = \int_0^{V(c_h - t_h, p) - V(c_l + t_l, p)} f(\theta)\, d\theta.$$

As $t_h > 0$, we have a reduction of high-productivity workers (increase in low-productivity workers) of $m - m(t_h) > 0$. By budget balance:

$$t_l = \frac{t_h m(t_h)}{n + m - m(t_h)}.$$

At equilibrium, the following condition must also hold:

$$\begin{aligned} 1 &= (n + m - m(t_h))(\frac{c_l + t_l}{p}) + m(t) x_e(p, t) + x_e(p) \\ \implies p &= (n + m - m(t_h))(c_l + t_l) + p m(t) x_e(p, t) + p x_e(p), \end{aligned}$$

giving the following equilibrium price:

$$p = \frac{1}{1 - m(t) x_e(p, t) - x_e(p)} (n + m - m(t_h))(c_l + t_l)$$

Remark. By market clearing:

$$\begin{aligned} (n + m - m(t)) \frac{c_l}{p} + m(t) x_e(p, t) + x_e(p) &= 1 \\ \implies (n + m - m(t)) c_l + p(m(t) x_e(p, t) + x_e(p)) &= p \\ \implies (n + m - m(t)) c_l &\leq p, \end{aligned}$$

which means that as n goes to infinity, price p goes to infinity.

Also, notice that both $m(t)$ and t_l are functions of t_h. Thus, the number of people working in the high job m is a function of t_h where $t_h = 0$ is the laissez-faire case. Also, notice that if $t_h \geq c_h - c_l$, no one works in high productivity jobs. Thus, $m(c_h - c_l) = 0$. Hence, t_h and $m(t)$ are bounded. Moreover, the dead-weight loss is given by:

$$DWL = \int_{m(t_h)}^{m} (c_h - c_l - \theta_\omega) f(\theta)\, d\theta. \tag{23}$$

Having set up the non-linear taxation regime, we now discuss the segmentation regime.

C.4.2 Segmentation

Under this setup, complete segmentation is a scenario wherein the planner caps the expenditure on essential goods at c_l. Thus, under this regime, people working in both jobs consume an equal amount of essential goods. However, high-productivity workers can potentially consume luxuries. Now, an individual with ability θ_ω works in a high-productivity job if and only if:

$$V(c_h, p) - \theta_\omega = u(\frac{c_l}{p}) + c_h - c_l - \theta_\omega \geq u(\frac{c_l}{p})$$
$$\leftrightarrow \theta_\omega \leq c_h - c_l$$

By market clearing,

$$1 = (n+m)\frac{c_l}{p} + x_e(p),$$

where $\frac{c_l}{p}$ and $x_e(p)$ is the consumption of workers and dons. The dead-weight loss is:

$$DWL = \int_{c_h - c_l}^{m} (c_h - c_l - \theta_\omega) f(\theta)\, d\theta$$

We now prove our main result. First, we show that when n is large, the planner primarily cares about the welfare of the poor. Under quasi-linear utilities and price inelastic essential goods, maximizing total welfare (almost) reduces to maximizing the consumption of the poor as their number increases.

Lemma 8.2. *Under the assumption of a price inelastic essential good, as n goes to infinity, the ratio of the marginal utility of the poor to that of the rich goes to infinity.*

Proof. The marginal utility of the essential good for the rich is λp, where λ is the marginal utility of income and p is its price. Thus, the ratio of the marginal utility of the poor to the marginal utility of the rich is $\frac{u'(\frac{c_l}{p})}{\lambda p}$.
Suppose this is bounded; this would mean:

$$\frac{1}{\lambda p} u'(\frac{c_l}{p}) \leq c \quad as \quad n \to \infty.$$

Price p is increasing in n. Therefore, as n goes to infinity, so does p. Hence;

$$\frac{1}{\lambda p} u'(\frac{c_l}{p}) \leq c \quad as \quad p \to \infty.$$

Under quasi-linear utilities, if this is true for $p \to \infty$, then it must be valid for all p. Therefore, it must be true for $p = \frac{c_l}{x_e(p)}$ as well, where $x_e(p)$ is the demand for the essential good. Thus, we obtain the following:

$$u'(x_e(p)) = u'(\frac{c_l}{\frac{c_l}{x_e(p)}})) \leq \lambda c \frac{c_l}{x_e(p)}.$$

From the first order conditions, $\lambda p = u'(x_e(p))$. Thus, $px_e(p) \leq c.c_l$, which means that $p(x_e(p))$-expenditure, is bounded above. However, we know that expenditure is an increasing and unbounded function of price, which is a contradiction.

Note this crucially depends on our assumption that $\sigma < 0$.

□

So, if n increases beyond a certain threshold, consumption of the poor always increases in t_h. Further, the planner almost only cares about the consumption of the poor; hence, optimally, t_h is set at the maximum level possible, *i.e.*, $c_h - c_l$. Moreover, at this level, $m(t_h) = 0$. Here, the dead-weight loss is at the highest level, *i.e.* no one works in high-productivity jobs. On the other hand, if we use segmentation, allowing the rich to spend only c_l on essential goods, we have the same allocation of essential goods but achieve greater production of luxury goods. We now state this formally as a theorem.

Theorem 9. *If the essential good is price inelastic, $\exists N \in \mathbb{N}$, such that, for $n \geq N$, complete market segmentation dominates non-linear labor taxation.*

First, we consider the optimal t_h under non-linear taxation. Under tax rate t_h, the measure of high-productivity workers in the economy is $m(t_h)$. Let x_e^{poor} be the consumption of the essential good by the poor, $x_e(p(t_h), t_h)$ by the rich, and $x_e(p(t_h))$ by the 'don.' Notice that taxes affect the rich both directly (through income) and indirectly (through price). For dons, the effect is only indirect. The consumption of the poor can be expressed as:

$$x_e^{poor} = \frac{1}{n}\left[1 - m(t_h)x_e(p(t_h), t_h) - x_e(p(t_h))\right]$$

Differentiating w.r.t. t_h:

$$\frac{dx_e^{poor}}{dt_h} = \frac{1}{n}\left[-\frac{dm(t_h)}{dt_h}x_e(p(t_h), t_h) - m(t_h)\frac{\delta x_e(p(t_h), t_h)}{\delta t_h} - (m(t_h)+1)\frac{\delta x_e}{\delta p}\frac{dp}{dt_h}\right].$$

As essential goods are inelastic,

$$\frac{\delta x_e}{\delta p}p + x_e(p) \geq 0 \implies |\frac{\delta x_e}{\delta p}| \leq \frac{x_e(p)}{p}.$$

Given that $\frac{\delta x_e(p(t_h), t_h)}{\delta t_h}$ and $\frac{dm(t_h)}{dt_h}$ are always less than equal to zero, we can lower bound the above term by

$$\frac{1}{n}\left[-\frac{dm(t_h)}{dt_h}x_e(p(t_h), t_h) - m(t_h)\frac{\delta x_e(p(t_h), t_h)}{\delta t_h} + (m(t_h)+1)\frac{x_e(p)}{p}\frac{dp}{dt_h}\right]. \quad (24)$$

Now, we show that, for large n, the above term is always positive. First, notice that:

$$t_l = \frac{t_h m_1}{n + m - m(t_h)},$$

and,

$$\begin{aligned} p &= (n + m - m(t_h))(c_l + t_l) + pm(t)x_e(p,t) + px_e(p) \\ \implies p &= (n + m - m(t_h))(c_l) + m(t_h)t_h + m(t_h)x_e(p,t_h)p + x_e(p)p. \end{aligned}$$

Hence:

$$\frac{dp}{dt} = \frac{dm}{dt}\left[t - c_l + px_e(p,t)\right] + m(t) + m(t)\left[p(\frac{\delta x_e(.)}{\delta p}\frac{dp}{dt_h} + \frac{\delta x_e(.)}{\delta t_h}) + x_e(p)\frac{dp}{dt}\right] + \frac{dp}{dt}\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]$$

$$\implies \frac{dp}{dt} = \left[1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right]^{-1}\left[\frac{dm}{dt}(t - c_l + px_e(p(t),t)) + m(t)(1 + p\frac{\delta x_e(p(t),t)}{\delta t})\right].$$

We can now compute $\frac{x_e(p)}{p}\frac{dp}{dt}$, which equals:

$$\left[1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right]^{-1}\left[x_e(p)\frac{dm}{dt}\left[\frac{t}{p} - \frac{c_l}{p} + x_e(p(t),t)\right]\right] +$$
$$\left[1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right]^{-1}\left[\frac{m(t)x_e(p)}{p}\left[1 + p\frac{\delta x_e(p(t),t)}{\delta t}\right]\right]$$

Now, putting this term back in 24, we get:

$$\begin{aligned} \frac{1}{n}\Bigg\{ &\left(-\frac{dm(t_h)}{dt_h}x_e(p(t_h),t_h) - m(t_h)\frac{\delta x_e(p(t_h),t_h)}{\delta t_h}\right) \\ &+ (m(t_h)+1)\left(1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right)^{-1}\left(x_e(p)\frac{dm}{dt}\left[\frac{t}{p} - \frac{c_l}{p} + x_e(p(t),t)\right]\right) \\ &+ \left(1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right)^{-1}\left(\frac{m(t)x_e(p)}{p}\left[1 + p\frac{\delta x_e(p(t),t)}{\delta t}\right]\right)\Bigg\}. \end{aligned}$$

As $\left[1 - (m(t)+1)\left[x_e(p) + p\frac{\delta x_e}{\delta p}\right]\right]$ is positive and $\frac{1}{p} \geq \frac{-\delta e}{\delta t}$, we know that 1st, 2nd and 4th terms are positive. We now compare the 1st and 3rd terms and show that their sum is positive. Adding, we get:

$$-\frac{dm(t_h)}{dt_h}x_e(p(t_h))\left[1 - \left[1 - m(t)x_e(p(t)) - \left[x_e(p) + p\frac{de}{dp}\right]\right]^{-1}\right]\left[\frac{t}{p} - \frac{c_l}{p} + x_e(p(t),t)\right]$$

Now, we know as $n \to \infty$:

$$\left[1 - m(t)x_e(p(t)) - \left[x_e(p) + p\frac{de}{dp}\right]\right] \to 1$$

and

$$\left[\frac{t}{p} - \frac{c_l}{p} + x_e(p(t), t)\right] \to 0.$$

Thus,

$$-\frac{dm(t_h)}{dt_h} x_e(p(t_h)) \left[1 - \left[1 - m(t)x_e(p(t)) - \left[x_e(p) + p\frac{de}{dp}\right]\right]^{-1}\right] \left[\frac{t}{p} - \frac{c_l}{p} + x_e(p(t), t)\right] \geq 0.$$

Further, as $n \to \infty$, $\frac{dx_e^{poor}}{dt_h} \geq 0$. Hence, t_h is set at the maximum level, *i.e.,* $c_h - c_l$. Consumption of the low productivity job holders (in this case, everyone except the dons) is given by $\frac{c_l}{p}$. Moreover, at this level of taxation $m(t_h) = 0$, and thus the dead-weight loss is given by:

$$DWL = \int_0^m (c_h - c_l - \theta_\omega) f(\theta)\, d\theta.$$

Under complete segmentation, the consumption of the low productivity job holders (again, everyone except the dons) is the same as the above, *i.e.,* $\frac{c_l}{p}$. However, the dead-weight loss is:

$$DWL = \int_{c_h - c_l}^m (c_h - c_l - \theta_\omega) f(\theta)\, d\theta.$$

As $c_h - c_l >> 0$, the dead-weight loss in the segmentary regime is much lower.

Remark. Notice that DWL under non-linear taxation is:

$$DWL = \int_0^m (c_h - c_l - \theta_\omega) f(\theta)\, d\theta$$

Hence, if the distribution of skills is very dense at the tail (close to zero), DWL is large. We believe this is analogous to the 'superstar phenomenon' because this means that the skill distribution in the economy is such that the number of very high-productivity workers (or superstars) is large. High taxes deter them from producing, which in turn, creates a lot of dead-weight loss. However, DWL in the case of segmentation is:

$$DWL = \int_{c_h - c_l}^m (c_h - c_l - \theta_\omega) f(\theta)\, d\theta,$$

which is unaffected by 'superstars' (the mass at or close to zero: the highest skilled individuals). Thus, when compared to non-linear taxation, MS performs better. A simple way to view this is that the efficiency gain under MS over taxation is:

$$Gain = \int_0^{c_h - c_l} (c_h - c_l - \theta_\omega) f(\theta)\, d\theta.$$

Hence, if the mass near zero increases, so does the efficiency gain.